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Decision Making Under Uncertainty

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Abstract: This report introduces concepts, principles, and approaches for addressing uncertainty in decision making. The sources of uncertainty in decision making are discussed, emphasizing the distinction between uncertainty and risk, and the characterization of uncertainty and risk. The report provides a brief overview of decision theory and presents a practical method for modeling decisions under uncertainty and selecting decision alternatives that optimize the decision maker's objectives. The decision modeling methods introduced in this paper are suitable for both data-rich and data-poor decision environments. This report describes how to analyze the sensitivity of a decision model to improve understanding of the decision problem and build confidence in the conclusions of an analysis. Principles of adaptive management and adaptive engineering are discussed from a decision analysis perspective. Examples are provided to demonstrate how these methods could be applied within the U.S. Army Corps of Engineers.

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Preface

This report presents concepts, principles, and approaches for addressing uncertainty in decision making and provides examples demonstrating how these methods might be applied. Preparation of this report was sponsored by the U.S. Army Corps of Engineers (USACE) Actions for Change Program, Theme 2: Risk-Informed Decision Making (now incorporated into the USACE Civil Works Campaign Plan, Goal 2). The Risk-Informed Decision Making theme was led by Dr. David Moser, USACE Institute for Water Resources. Publication of this report was jointly sponsored by the Dredging Operations and Environmental Research Program, Dr. Todd S. Bridges, Program Manager, and the Flood and Coastal Storm Damage Reduction Program, Bill Curtis, Program Manager.

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1 Introduction

Why uncertainty matters

Uncertainty is often an unavoidable factor in making risk management decisions. The U.S. Army Corps of Engineers (USACE) makes risk management decisions on a routine basis. For example, these decisions are made in the process of managing the nation's navigation system, selecting flood risk management alternatives, designing and operating dams and reservoirs, and selecting ecosystem restoration strategies. Uncertainties concerning the performance of navigation, flood control, and ecological systems arise from inherent variability in the processes that affect those systems (e.g., weather patterns, economic trends, etc.) and incomplete knowledge of these processes.

A sound approach to rational decision making requires a decision maker to establish decision objectives, identify alternatives, and evaluate those alternatives with respect to those objectives. Often, there is much uncertainty in forecasting the outcomes of alternatives, particularly when decisions are complex. Such decisions are said to be risky because the outcome following a choice may result in a potential loss, including lost opportunities or sub-optimal outcomes. The purpose of this report is to present methods and approaches that enable a decision maker to make choices under uncertainty with confidence. The methods described in this paper take into account uncertainty in the forecasted decision outcomes and the decision maker's individual preferences with respect to risk. The methods presented in this paper do not guarantee that the outcome of a particular risky decision will be optimal or "good," but only that the decision will be rational in the face of uncertainty and that repeated application of these methods will maximize the decision maker's welfare over the long run.

Risk-informed decision making

A variety of methods might be used to try and overcome the challenges that uncertainty poses for decision makers. On one end of the spectrum are probabilistic risk and decision analysis methods. On the other end of the spectrum are ad-hoc methods (those developed specifically for a particular decision) and even intuition, if that can be called a method. In contrast to probabilistic risk and decision analysis, ad-hoc methods and

intuition are unlikely to provide a defensible basis for decision making. This is particularly true in cases where decision makers hold the public trust, which is the case when potential losses would be distributed across a population that may have had little or no input into a decision process. In such cases, the use of a rational and rigorous approach to decision making is needed - both to protect the decision maker and to protect the public.

Probabilistic risk and decision analysis is the most (and some would say "the only") rigorous engineering approach to difficult decision-making problems involving uncertainty. This report provides a brief introduction to these concepts and analytical methods and demonstrates the potential value of these methods using examples. This report stresses the development and use of decision models to explore the sensitivity of a decision and its potential outcomes. Like all forms of modeling, decision modeling requires skill and even some degree of artistry. Throughout the process, it is important to remember that the decision modeling activity is not about "getting the answer." Rather, it is ultimately about learning more about the decision problem itself. A good decision model will lead the decision maker to a more sophisticated understanding of his particular decision problem by enabling him to pick the problem up and explore it or inspect it from a variety of different angles. This exploration of the decision problem through sensitivity analysis is eventually what builds confidence in the recommendation on the part of decision makers and stakeholders.

It is important to note that the decision modeling described in this paper is not a substitute for a decision-making process or a decision maker. These methods help to provide structure to the relevant information and to increase the level of understanding about the choices that are being made. These methods are not a substitute for the decision maker because, ultimately, the decision maker's values must be taken into account. In this report, the emphasis on values is with respect to the level of risk aversion that a decision maker may have. Two decision makers using the same decision model can reach different conclusions depending upon risk preferences.

Decision analysis methods also have much to offer decision makers and stakeholders who must construct preferences or achieve consensus through common understanding of a decision problem. Decision analysis methods assist decision makers and stakeholders to construct their preferences by learning more about the alternatives, the consequences,

and their probabilities. Decision analysis methods also assist decision makers and stakeholders to reach consensus by revealing where differences in stakeholder opinions and values really matter (or don't matter) and facilitating negotiation and focusing discussion on the critical issues.

Organizing the analysis

In the context of using risk and decision analysis to support decision making, four major steps can be considered:

1. Framing the decision problem;
2. Modeling the decision;
3. Analyzing and interpreting the results;
4. Communicating the results to decision makers.

The focus of this report is primarily directed at steps 1 through 3. However, communicating the results to decision makers is equally important. A thorough and well executed analysis that is poorly communicated will not provide decision makers with the understanding they need to address uncertainty with confidence.

Considerable effort may be required to implement some of the analyses described in this report. This effort to provide decision makers with a comprehensive understanding of the decision problem and the relevant uncertainties should be justified by the potential costs of making a poor decision. This goal can be met without producing the perfect analysis. Investments in analyzing a decision problem should reflect the magnitude of potential losses associated with choosing a sub-optimal alternative. Higher opportunity costs justify greater investments in decision analysis.

In addition to the question of how much to invest in analyzing the problem, the timing of that investment is also critical. The risk and decision analysis approaches and practices described in this report will yield the greatest value to decision makers when they are incorporated at the very beginning of the project. How the decision problem is framed at the beginning of a project will determine what types of analysis should or can be performed as well as what data will be needed to support the analysis and the ultimate decision.

Quantitative vs. qualitative analysis

The methods described in this report are geared towards the quantitative analysis of decision problems. Some elements of problems are more amenable to quantification than others. However, there are often ways of addressing qualitative issues quantitatively and the only barrier to implementing these methods is often a lack of awareness on the part of analysts that these methods exist. For example, risk preferences among stakeholders have often been ignored or treated qualitatively. This report discusses the utility function, which incorporates a risk tolerance parameter that can be used to reflect a decision maker's preference with regard to accepting risk. As another example, consider ecological outcomes of decisions that are often described as being "non-monetizable." Methods exist to quantify and monetize the benefits and costs of any ecological decision outcome, although it can sometimes be very difficult to do so. Sometimes, it may in fact be truly impractical or impossible to address qualitative issues in a decision problem quantitatively. It is beyond the scope of this report to address strategies or techniques for addressing qualitative elements or issues in decision analysis unless they can be described quantitatively. However, a credible decision will consider all of the important components of a decision problem, not just those that are conveniently quantified.

Organization of this report

This report is organized as follows. Chapter 2 addresses the concepts of uncertainty and risk emphasizing the different sources of uncertainty, the distinction between uncertainty and risk, and the characterization of uncertainty and risk. Chapter 3 defines "decision making under uncertainty" and introduces decision theory and decision analysis. Value functions and utility functions that capture information about a decision maker's attitudes toward risk are central to this discussion. Chapter 4 introduces a practical approach to modeling decisions under uncertainty. This section walks through the steps of developing a decision model and applying the concepts discussed in Chapters 2 and 3. Chapter 5 presents a simple example of decision analysis to illustrate use of the methods. Chapter 6 discusses how probabilistic decision analysis can be used to implement adaptive management and adaptive engineering principles. Chapter 7 describes the application of these methods to a dredging decision to demonstrate how the decision making under uncertainty methods might be used to address day-to-day decision problems in USACE. Chapter 8 discusses practical aspects of implementing the techniques discussed in this report.

2 Uncertainty and Risk

What is uncertainty and what are the types of uncertainty?

Uncertainty is a lack of knowledge. Among the various fields that are concerned with uncertainty, there is no common agreement on the terminology, definition, or classification of uncertainty. Several useful typologies exist (Ascough et al. 2008). Typologies are intellectual constructs; therefore, it is appropriate to choose the typology that is most useful given the purpose of the work. This report adopts a typology that has been widely used and has proven to be a useful way of thinking about uncertainty in the context of quantitative analysis.

Uncertainty can be classified either as input uncertainty or model uncertainty. Input uncertainty arises from a lack of knowledge about the true value of quantities used in analyzing a decision. Often, these quantities are found in scientific models that are used to support a decision, such as hydrologic and environmental models. Model uncertainty is uncertainty about the form of the model used to support the decision. In other words, model uncertainty is uncertainty about what variables, assumptions, and functions best characterize the processes being modeled. In practice, model uncertainties are much more difficult to deal with than input uncertainties because they require the analyst to propose and evaluate competing models (Casman et al. 1999). The discussions and examples in this report emphasize how to address the problem of input uncertainty. However, this does not imply that model uncertainty is less important and the techniques that might be used to address model uncertainty are often similar to those discussed in this report.

Input uncertainty is often attributed either to heterogeneity in nature (natural variability) or to a lack of knowledge.[1] If the uncertainty in an input variable is attributed to natural variability, then that input variable cannot be known precisely because the true value of that quantity in nature varies spatially and/or temporally. Natural variability cannot be controlled or eliminated; therefore, uncertainty attributed to natural variability cannot

[1] Uncertainty attributed to natural variability is called aleatory uncertainty. Uncertainty attributed to a lack of knowledge is called epistemic uncertainty.

be reduced by obtaining more information. In contrast, knowledge uncertainty can always be reduced by obtaining more information, although it may be very difficult, expensive, or physically impossible to do so in practice. Input uncertainty may be described as being attributed to either heterogeneity in nature or a lack of knowledge, but model uncertainty is always attributed to a lack of knowledge. Input uncertainty can usually be attributed to both natural variability and a lack of knowledge.

Uncertainties can be assessed through observations and described in terms of frequencies and probability distributions. However, risk and decision analysts are often concerned with quantities that cannot be observed, measured, or counted. Limits on the ability to observe quantities in nature may arise in practice because it is too costly, time consuming, or technologically infeasible to make the observations, or in principle because that quantity in which we are interested, such as the probability of a rare event or condition occurring in the future, cannot be observed. Therefore, most risk and decision analysts adopt a Bayesian view of probability in which probability describes an individual's "degree of belief." This is also known as subjective probability.

The Bayesian view of probability holds that probability measures the confidence that an individual has in the truth of a particular proposition. For example, an individual might assess the probability that it will rain the following day using information about the extent of cloud cover on the evening prior to that day. In contrast to the Frequentist view of probability, which holds that probability can only be assessed using information about the frequency of an event or condition, subjective probabilities are not so constrained. Subjective probabilities can also be assessed without reference to whether or not the events are determined or somehow known by others (Miles 2007). From this perspective, uncertainty describes the state of the observer in relation to that which is being observed, rather than the state of that which is being observed. Savage (1954) showed that subjective probabilities can conform to Kolmogorov's axioms of

probability[1] and, therefore, frequentist theory can be extended to analyze degree of belief.

The extension of frequentist probability theory to the analysis of uncertainty in "things" that cannot be observed or counted has been contentious and problematic. However, subjective probability assessments and distributions are essential tools for risk and decision analysts because the observations necessary to make objective probability assessments are not always possible or feasible. Subjective probabilities should be based on available evidence and previous experience with similar events, they must be plausible, and they must conform to Kolmogorov's axioms (Morgan and Henrion 1990, Garvey 2008). The invitation to use subjective probabilities must not be seen as an invitation to be arbitrary or otherwise to avoid or neglect evidence. Subjective probability assessments must be founded on some form of defensible reasoning or verifiable experience. If it is perceived that probabilities are based on limited insight and experience, they can undermine an analysis.

Subjective probabilities are not appropriate to describe volitional uncertainty, which is uncertainty on the part of the decision maker about future preferences or actions. However, decision makers can assess subjective probabilities regarding what somebody else might do (Bedford and Cooke 2001, p. 35). Subjective probabilities should not be considered uncertain because, by definition, a decision maker's beliefs must be known to himself (De Finetti 1974). However, objective probabilities (*i.e.*, frequencies) - those known from observations - can be uncertain. A Bayesian's subjective probability distribution about an empirical quantity should converge with a frequentist's objective probability distribution as the evidence used in developing the two distributions converges (Morgan and Henrion 1990).

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[1] The axioms of probability express what must be true for the theorems of probability calculus to hold. The theorems are discussed by Garvey (2008). The axioms can be summarized as follows:

1. The probability of any event *A* in a set of events, Ω, is a non-negative number in the interval zero to one: $0 \le p(A) \le 1$.
2. The elements of the set Ω are collectively exhaustive such that, the probability of the occurrence of an element of the set is one: $p(\Omega) = 1$.
3. For a mutually exclusive set of $k = \{1,2,3,\ldots,K\}$ events, the probability of an aggregate of those events is the sum of the probabilities of the individual events:

$$P\left(A_1 \cup A_2 \cup \ldots A_i \ldots \cup A_N\right) = \sum_k^K p\left(A_k\right)$$

Other authors have noted that the use of subjective probabilities is often complicated by ambiguity in the quantity for which uncertainty is being assessed. Ambiguity is not a type or source of uncertainty because it can be removed by careful definition of the quantity in question (Bedford and Cooke 2001). Morgan and Henrion (1990) describe a simple "clarity test" for evaluating whether a quantity is sufficiently well-specified to assess valid subjective probabilities. "Imagine a clairvoyant who knows all facts about the universe, past, present, and future. Given the description of the event or quantity in question, could this clairvoyant state unambiguously whether the event had (or will) occur, or could this clairvoyant give the exact numerical value of the quantity? If so, then the quantity or event is sufficiently well specified." These authors attribute the clarity test to Howard and Matheson (1984).

Uncertainty about the true value of an input variable can be described in several ways. Frequency distributions, statistical variances, coefficients of variation, confidence intervals, and probability distributions are commonly used to describe the uncertainty in quantities. Of these, probability distributions offer the most complete and compact form of representation. Figure 1 illustrates three ways to characterize uncertainty in a random variable. Figure 1(a) is a histogram, which is useful in describing uncertainty in discrete random variables. Figure 1(b) is a probability density function (PDF), which is a particular class of functions that possess the property that integration of that function over all possible values yields one. The PDF is useful for describing uncertainty in continuous random variables. Integration of the PDF yields a cumulative distribution function (CDF), shown in Figure 1(c). The CDF gives the probability that *x* is less than some amount.

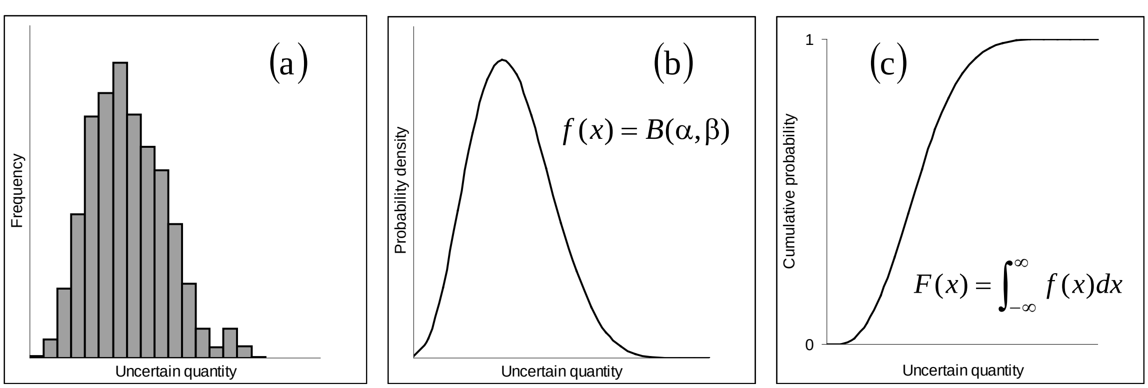


Figure 1. Three methods of characterizing uncertainty in a random variable.

What are sources of uncertainty in decisions?

Uncertainty in decision making can be attributed to uncertainty in the input variables and models used in simulating decision outcomes. For example, life-cycle decisions that maximize the net present value of national economic development (NED) benefits over a 50-year planning horizon may be sensitive to the models and input values used to forecast economic growth, population growth, and investment. Similarly, ecological restoration decisions may be based on forecasts of ecological effects, which must be modeled. Modelers usually attach a considerable amount of uncertainty to input values used in these models. Uncertainty in the outputs of models used to simulate decision outcomes depends upon the sensitivity of those models to the uncertain inputs and the amount of uncertainty in those inputs. This report does not discuss the propagation of uncertainties in models.

While it is important to know what the sources of uncertainty in a decision are, it is just as important to know where the sources of uncertainty are not. As discussed above, there can be no uncertainty about subjective probabilities or volition. Similarly, there can be no uncertainty in value parameters such as the discount rates that reflect a decision maker's rate of time preference for money or weights used in multi-attribute value functions that reflect a decision maker's willingness to trade off among components of value. The underlying assumption of decision analysis is that a decision maker is known and knows his or her values/preferences. Analysts who are preparing decision analyses for third parties or collective decision makers may be unclear about what value parameters to use. An important point emphasized in this report is the need for analysts to report the sensitivity of their decision models to assumptions about subjective probabilities and value parameters. Methods of doing this will be demonstrated through example later in this report. Doing so helps decision makers focus discussion on the critical regions of the decision landscape where uncertainties matter because they might change the decision and set aside differences in opinion when those differences wouldn't change the optimal decision.

There can be no uncertainty in the decision frame. The decision frame is the set of decision alternatives and objectives. The ability to develop frames that provide a meaningful simplification of more complex problems requires skill in decision modeling and a deep understanding of the decision problem. The set of alternatives is usually a relatively small subset

from the decision space, the set of all possible decision alternatives. A decision objective provides a basis for evaluating each alternative. For example, the objective of a decision may be to maximize the net present value of national economic development benefits over the planning horizon. At the beginning of a decision-making process, there may be ambiguity about what decision frame to use, but the decision maker must have clarity on the decision frame before an analysis of alternatives begins. The decision-making process often involves an iterative learning process through which decision makers and analysts refine and update the decision frame. It is important to allow flexibility in the decision-making process so that this can occur.

What is risk and what is the distinction between risk and uncertainty?

A risk is a potential adverse consequence that may or may not be realized in the future. An adverse consequence is a loss of some sort. A decision maker faces a risk if the outcome of a decision is uncertain and may be adverse. In a paper that was published in the first issue of the journal *Risk Analysis*, Kaplan and Garrick (1981) suggested that risk can be fully defined by a set of three things, including: 1) a set of mutually exclusive and collectively exhaustive scenario conditions under which the possible outcomes may be realized, 2) a set of outcomes for each possible scenario, and 3) a probability of occurrence for each possible scenario. Using this definition, risk can be described using a loss-exceedance curve. In a loss-exceedance curve, scenario outcomes involving potential losses are plotted on the *x* axis and the probability of exceeding those losses is plotted on the *y* axis (Figure 2). The loss-exceedance curve is sometimes called a risk curve.

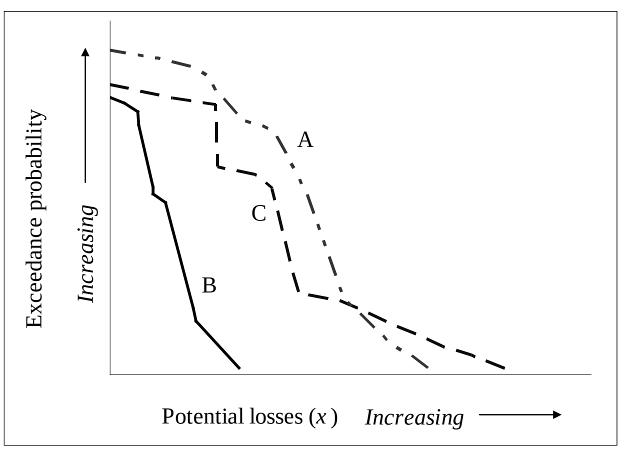


Figure 2. Three risk curves.

Figure 2 illustrates three risk curves, which could represent the potential losses associated with three decision alternatives (A, B, and C). The *y*-intercept gives the probability that the costs associated with choosing an alternative will exceed the benefits of that alternative. In Figure 2, Alternative C entails the largest potential losses. Alternatives A and C are riskier than Alternative B because these alternatives lead to larger losses with higher probabilities. It is important to note that these risk curves by themselves do not provide the decision maker with sufficient information to choose among the three alternatives. A decision maker also needs information on the potential benefits of each alternative and their probabilities. An understanding of the decision maker's attitudes toward accepting risks is also needed.

What is risk analysis?

Risk analysis is an interdisciplinary field of study. Individuals who practice risk analysis attempt to quantify, manage, and understand financial, economic, human health, and environmental risks. The field of risk analysis includes risk assessment, risk management, and risk communication (Pate-Cornell and Dillon 2006). A risk assessment provides the answer to three questions: 1) What could go wrong and how could it happen? 2) How likely is it to happen? and 3) What are the consequences should it happen? Risk assessments can be either qualitative or quantitative, but effective use of qualitative risk assessment techniques generally requires a good understanding of quantitative risk assessment techniques, the objective of which is to obtain a distribution of probabilities over potential losses. Risk management is a process of managing the exposure to risks so that economic benefits are maximized. Among other things, risk management includes formulating, evaluating, selecting, and implementing risk management alternatives. Risk communication involves communicating information about risks, emphasizing that communication is a two-way interactive process involving listening and learning from stakeholders as well as presenting information to stakeholders.

Risk management decisions need to be made within the context of the social perception of the particular risk at issue. An individual's perception of a risk and the collective perceptions of society affect the extent to which society accepts or tolerates risks. Acceptance can be described as a function of the extent to which the exposure is voluntary, the dread associated with the outcome, knowledge about the processes generating the outcomes, the extent to which the individual can exert control over the

outcome, the potential benefits that acceptance of the risk provides, the number of deaths caused in a typical year, and the number of deaths caused in a disastrous year (Starr 1969, Fischoff et al. 1978, Slovic 1987). Society adopts different standards for managing and accepting these different risks.

What risks are associated with decision making?

This report is concerned particularly with choosing alternative courses of action in the face of uncertainty about the outcomes that will be realized as a result of those actions. These uncertainties are attributed to uncertainties in the inputs and model forms used in forecasting the outcomes. Alternatives are risky if a decision maker (an individual, corporation, or society) could incur a financial or economic loss as a result of choosing that alternative. Financial losses are distinguished from economic losses because the former are limited to a comparison of project revenue and expenses, whereas the latter involve an evaluation and comparison of a much broader range of benefits and costs, including those that might be classified as social or environmental.

Risk-informed decisions are based on information about uncertainty in the outcomes. Decisions themselves are risky if the decision maker could sustain an opportunity cost as a result of choosing an alternative that leads to a sub-optimal outcome. Opportunity costs are economic costs that may be realized when resources are invested in one project and it turns out that greater net benefits could have been realized by investing those funds in an alternative project. Decision analysis should reveal the potential opportunity costs associated with an alternative. This is accomplished through sensitivity analysis.

3 Decision Theory and Decision Analysis

What is decision making under uncertainty?

A decision is a choice between two or more courses of action. Decision making under uncertainty is the act of choosing between two or more courses of action when the outcomes of those actions are uncertain. Figure 3 introduces the decision tree, a diagram that helps to structure the decision when uncertainties are present. This particular decision is between a "*sure thing*" [A_1] and a lottery [A_2]. The sure thing rewards individuals with a guaranteed amount, x_1, while the outcome of the lottery is uncertain. Under the lottery, the individual faces a chance to receive a larger reward with probability p or no reward with probability 1-p. The expected value of a lottery is the probability weighted sum of possible outcomes: $E[A_2] = px_2 + (1-p)x_3$.

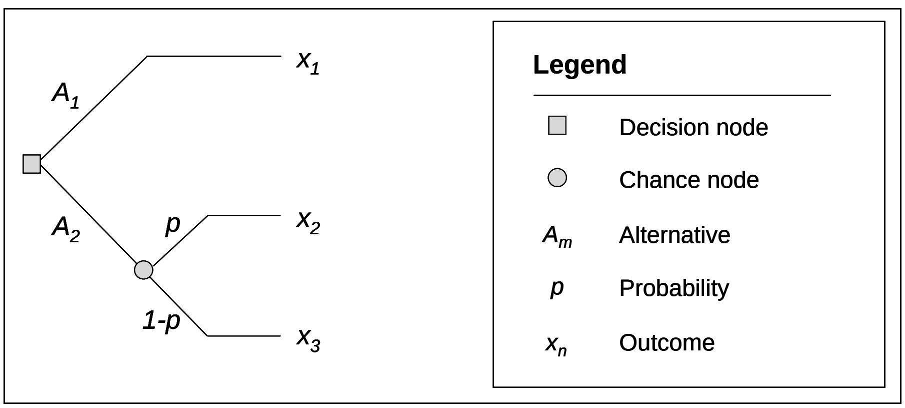


Figure 3. A decision tree showing two alternatives and three outcomes. This figure illustrates a choice between A_1, which has a certain outcome, x_1, and A_2, a lottery, which has one of two possible outcomes, x_2, which occurs with probability p, or x_3, which occurs with probability $1-p$.

What is decision theory?

The study of how individuals make decisions when faced with a choice that has an uncertain outcome such as that illustrated in Figure 3 is known as decision theory. Prior to 1713, the conventional wisdom was that individuals choose actions based on the expected value of the outcomes of those actions. This notion was challenged in the St. Petersburg paradox, which proposed a game in which a fair coin is tossed until it comes up heads. The payoff is 2^n, where n is the number of times the coin is tossed. Mathematically, this game has an infinite expected payoff. The paradox

was this: If people choose actions that maximize the expected value of the outcome, then why are most individuals not willing to pay any amount of money to play this game that has an infinite expected payoff?

The St. Petersburg paradox was resolved in 1738 by Daniel Bernoulli, who introduced the concept of a value function to describe the diminishing marginal value of wealth. He proposed that an individual's level of satisfaction can be described as a function of wealth, that there is a diminishing marginal satisfaction associated with increasing wealth (implying that satisfaction has upper and lower bounds), and that individuals make choices to maximize expected levels of satisfaction rather than expected value of wealth.

Defining the outcomes of a decision in terms of Bernoulli's value function did not resolve all of the issues. Bernoulli's theory indicated that people should be indifferent between the sure thing and a lottery with an identical expected outcome. However, it was widely observed that individuals who faced this particular decision chose the sure thing over the lottery.[1] In 1947, von Neumann and Morgenstern addressed this issue by extending the concept of the value function to incorporate attitudes toward both wealth and risk. The function that measures attitudes toward wealth and risk is called a utility function. Von Neumann and Morgenstern also developed a set of axioms that explained what a decision maker *must* believe in order for the theory of expected utility maximization to be true. An axiom is a statement that is so apparently true on its face that it requires no proof. The axioms are:

1. **Ordering:** The decision maker is capable of expressing whether he prefers outcome A over outcome B, or whether he is indifferent between the two outcomes. This means that the decision maker knows his preferences over outcomes.
2. **Transitivity:** If the decision maker prefers outcome A to outcome B, and outcome B to outcome C, then the decision maker prefers outcome A to outcome C.

[1] This choice is illustrated in Figure 3. An individual is compelled to choose between a guaranteed outcome, $x_1>0$, and a lottery that has an outcome of either $x_2 = 0$ with probability p or x_3 with probability $1-p$. If x_3 and p are chosen so that the expected value of the lottery is equal to x_1, most individuals choose the sure thing over the lottery.

3. **Continuity:** Given a choice between a sure thing with outcome x_1 and a lottery with two outcomes, x_2 and x_3, there exists a probability of realizing outcome x_2 such that the decision maker is indifferent between accepting the lottery and accepting the sure thing.
4. **Substitution:** If a decision maker is indifferent between two outcomes, then one outcome can serve as a substitute for the other outcome. If a decision maker is indifferent between an outcome and a lottery, the lottery can substitute for the event that leads to that outcome.
5. **Monotonicity:** If two lotteries yield the same outcomes, then given a choice between the two lotteries, a decision maker prefers the lottery with the higher probability of the preferred outcome.
6. **Reduction of compound events:** Complex events (events consisting of a mixture of lotteries) can be reduced to a simple event using standard probability manipulations without affecting a decision-maker's preferences.
7. **Invariance:** Preference among uncertain events can be established by knowing the utility of the outcomes and their probabilities.
8. **Finiteness:** No outcome is infinitely bad or infinitely good.

Von Neumann and Morgenstern (1947) assumed that all probabilities were objectively determined. The revolutionary contribution of Savage (1954) was to extend the von Neumann and Morgenstern expected utility maximization model by introducing subjective probabilities. The subjective expected utility maximization model is the foundation of modern decision analysis. There are two major branches in the field of decision analysis. Normative decision theory studies how decisions should be made and focuses on developing models of rational choice, algorithms, and analytical tools that lead decision makers to choices that are consistent with the axioms of rationality. Descriptive decision theory studies models that explain how people make judgments and decisions in "real life," often through observations that are made in controlled laboratory settings. The two fields are fairly distinct. Descriptive decision theory is largely dominated by social, behavioral, and cognitive psychologists. Normative decision theory is largely dominated by economists, operations researchers, engineers, and statisticians.

Laboratory studies have shown that individuals often violate the axioms of rationality when making decisions under uncertainty. The systematic description of such observations may lead to the formation of a paradox. For example, Maurice Allais and Daniel Ellsburg each identified paradoxes

that are still unresolved (Nau 2007). Some studies have attributed deviations from rationality to the use of heuristics to assess probabilities. A heuristic device is a rule of thumb, a simplification, or an educated guess that reduces the cost of searching for an optimal solution or the precise answer to a complex problem. Tversky and Kahneman (1974) described three heuristic devices that are commonly used in assessing probability and concluded that these heuristics introduced systematic bias into probability estimates, yielding decisions that do not conform to rationality.

- **Representativeness** is a device by which the probability of an object's membership in a larger group is based on the degree of similarity between that object and the larger group. Error arises from ignoring basic factors that should logically determine those probabilities, such as base rate, sample size, predictability, etc.
- **Availability** is a device by which the probability of an event or the frequency of an object's occurrence is based on the ease with which one can recall a similar event or imagine its occurrence. Errors arise because frequency and probability are determined by factors other than one's ability to recall events or imagine things.
- **Adjustment and anchoring** is a device by which an initial estimate of probability is based on a generalization and then adjusted to account for conditional factors. Errors arise because individuals tend to underestimate the level of adjustment that is required, which leads to "anchoring" on the initial value.

The fact that individuals have been shown to deviate from rationality when making decisions involving risk does not undermine the logic of the axioms and does not diminish their value as a normative basis for prescriptive analysis when uncertainties are present. The axioms continue to provide a well understood and widely accepted standard for evaluating preferences over outcomes and subjective expected utility maximization provides a normative and defensible standard for making decisions under uncertainty.

What is decision analysis?

Decision analysis is a collection of analytical problem-solving techniques for probability and preference assessment and decision modeling. Howard (2007) describes it as a logical procedure for balancing the factors that influence a decision. The procedure incorporates information about uncertainties, values, and preferences in a basic structure to model the decision. The essence of the procedure is the construction of a structural

model of the decision in a form that is suitable for computation and manipulation. The realization of the structural model is often a set of computer programs. The goal of practicing decision analysis is to help decision makers make defensible decisions that are rational in the sense of utility theory. Decision analysis came about from the merging of systems analysis and statistical decision theory, and it draws on the disciplines of mathematics, economics, behavioral psychology, and computer science (von Winterfeldt and Edwards 2007).

Decision analysis methods are specifically founded on normative decision theory or support the application of those techniques. Examples include means-ends networks and objectives hierarchies for structuring decision objectives, consequence tables for evaluating multiattribute value or utility functions, decision trees and influence diagrams for decision making under uncertainty, and event trees, fault trees, and belief networks for probabilistic inference (von Winterfeldt and Edwards 2007). Applications of decision analysis techniques are prescriptive because they indicate what a decision maker should do if he accepts the axiomatic foundations of decision theory. Methods such as analytical hierarchy process (AHP) (Saaty 1980), Dempster-Shafer theory (Dempster 1968, Shafer 1976), and fuzzy sets (Zadeh 1965) do not necessarily lead decision makers to rational choices and are therefore excluded from the field of decision analysis (Howard 2007, Lund 2008).

Although decision analysis may be informed by the results of a risk analysis, decision analysis is distinguished from risk analysis by several features. Decision analysis yields a choice among alternative courses of action while risk analysis yields a probability distribution over potential outcomes. Whereas risk analysis can be completed without the input of the decision maker, decision analysis cannot (Pate-Cornell and Dillon 2006). In decision analysis, the decision maker is known and must be explicit about his preferences by providing a value or utility function. Therefore, decision analysis does not relieve the decision maker of responsibility for making a decision. Faced with the same decision and armed with the techniques of decision analysis, two decision makers could choose two different courses of action. Both could be equally valid. How could this be? The answer is that the decision makers have different values. Decision analysts use value functions to describe a decision maker's attitudes toward wealth.

What is a value function?

A value function is a real-valued mathematical function that is defined over an attribute scale and describes how much value a decision maker realizes from achieving different attribute levels. A value function is illustrated in Figure 4. The y-axis is a value scale from 0 to 1 and the x-axis is an attribute scale, which represents wealth. Attributes may be monetary or non-monetary. Attribute scales may be either cardinal or ordinal. Cardinal scales are interval or ratio scales that express the strength of preference between two outcomes. Ordinal scales are scales that only express a preference order, but not the strength of preference among attribute levels. Attribute scales may be either continuous or discrete.

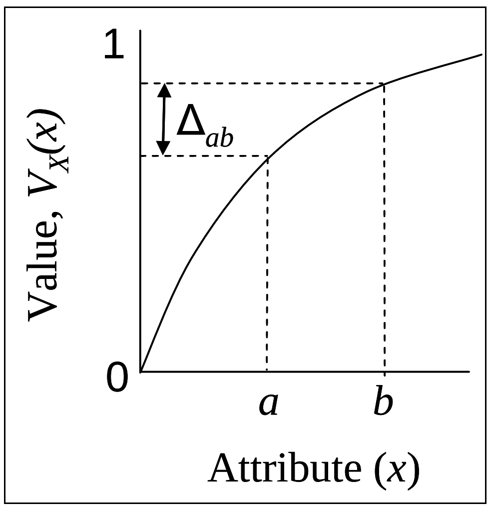


Figure 4. A monotonically increasing value function $V_X(x)$ over outcomes expressed in terms of an attribute x. The x-axis is sometimes defined as wealth.

The value function ranges from 0 to 1, with 0 representing the value of the least preferred outcome and 1 representing the value of the most preferred outcome. All value functions are monotonic, but not all value functions are increasing. Monotonically decreasing functions express the idea that more of an attribute is less desirable. This report assumes that value and utility functions are single attribute and do not deal with the case of multi-attribute functions. Multi-attribute value and utility functions are discussed at length by Keeney and Raiffa (1993).

The value increment is the change in value associated with an increase in the attribute level. In Figure 4, the value function is concave and the increment between a and b is: $\Delta_{ab} = V_X(b) - V_X(a)$. This value increment is

diminishing as x increases. This is a common characteristic of value functions that expresses the concept of diminishing marginal value of wealth.

What is a utility function?

A utility function expresses an individual's diminishing marginal value of wealth simultaneously with his risk attitudes, which are his attitudes toward the magnitude of prospective losses in relation to wealth. The utility function is a real-valued mathematical function that is defined over an attribute scale and describes how much utility (satisfaction) a decision maker realizes by achieving various attribute levels. The definition is similar to that of a value function, but the difference between a utility function and a value function should become apparent. Figure 5(a) illustrates a utility function. The y-axis is a utility scale, measured in units of *utils*, which are arbitrary units of satisfaction. As with the x-axis of the value function, the x-axis of the utility function may be cardinal, ordinal, continuous, or discrete.

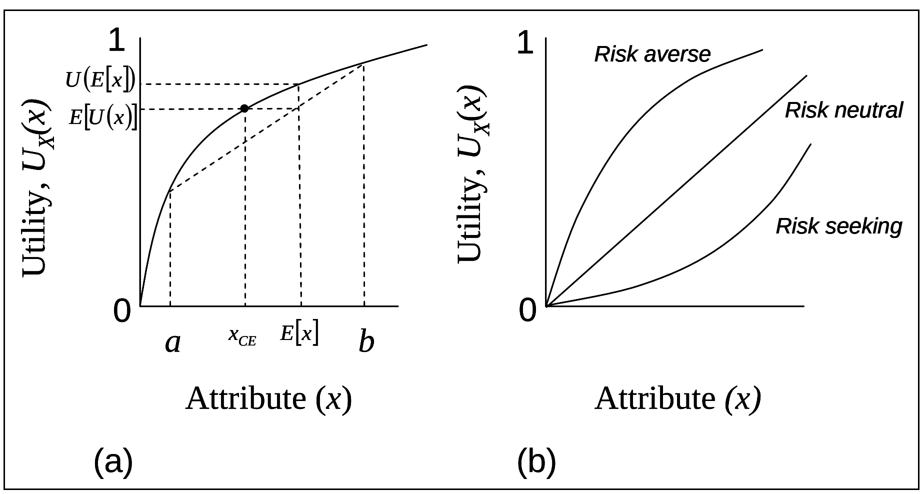


Figure 5. A single, risk averse utility function (a) and three alternative utility functions illustrating three risk attitudes (b).

The expected outcome of a lottery with two possible outcomes, a and b, is the probability weighted sum of the two possible outcomes: $E[x] = pa + (1-p)b$. The utility of the expected outcome is: $E[U(x)] = pU(a) + (1-p)U(b)$. Figure 5 shows that, for this particular utility function, the utility of the expected outcome of the lottery is greater than the expected utility of the lottery. The **certainty equivalent**, x_{CE}, is the amount of the certain outcome (x_1 in Figure 5) that would make the decision maker indifferent between the certain outcome and the lottery.

The **risk premium** is the difference between the expected outcome and the certainty equivalent: $R = E[x] - x_{CE}$. It is the minimum amount that a decision maker would have to be compensated to accept a lottery over a sure thing, or the amount the decision maker would be willing to pay to avoid choosing the lottery.

Figure 5 shows three possible risk attitudes. Risk attitudes describe a decision-maker's preferences with respect to accepting risk. Risk attitudes can be risk averse, risk neutral, or risk seeking.

- **Risk averse** behavior is described by a concave utility function and means that the decision maker would have to be compensated to voluntarily accept a lottery in a choice between a sure thing and a lottery with equal expected payoffs. This is the most common attitude toward risk encountered among individuals. The function has the property that $E[x] > x_{CE}$.
- **Risk neutral** behavior is described by a linear utility function. The decision maker is indifferent between a lottery and a sure thing that have equal expected payoffs. This function might be used to describe the behavior of insurers and investment banks. The function has the property that $E[x] = x_{CE}$, or $E[U(x)] = U(E[x])$.
- **Risk seeking** behavior is described by a convex utility function. This function suggests an individual would be willing to pay for the exposure to an uncertain outcome that has the same expected outcome as an alternative certain outcome. The function has the property that $E[x] < x_{CE}$. Risk seeking utility functions might be used to describe gambling behavior or utility under debt.

An implicit assumption of the functions illustrated in Figures 4 and 5 is that more is better. This is generally true with regard to money, environmental quality, health, crop yields, and many other goods. However, it is also possible to develop utility functions for "economic bads," which are attributes for which more is worse. Examples of economic bads include pain and suffering, property damages, and economic or financial losses, etc.

Some individuals who study utility have reasoned that an individual's utility function may change over levels of wealth. The Markowitz utility function, illustrated in Figure 6(a), is defined in reference to current wealth (Markowitz 1952). This utility function exhibits risk aversion immediately below current wealth and risk-seeking behavior immediately

above current wealth. However, as one moves further from the current wealth position, risk attitudes change. If one moves far enough below the current wealth position, risk-seeking behavior sets in. If one moves far enough above the current wealth position, risk-averse behavior sets in.

Kahneman and Tversky (1979) and Tversky and Kanheman (1992) developed prospect theory to explain why some individuals may have utility functions that differ from what is considered to be rational. These authors found that individuals tend to value changes in wealth relative to a reference point, rather than valuing net wealth, and that "losses loom larger than gains." Individuals exhibit risk-seeking behavior with respect to potential gains, but risk-averse behavior with respect to potential losses (Figure 6(b)). [1] Prospect theory attributes this behavior to: 1) judging the value of decisions in terms of deviations from a reference point rather than in terms of net wealth, 2) valuing losses differently than gains; and 3) placing too much importance on low probability outcomes and too little importance on moderate and high probability outcomes, rather than placing importance in proportion to the probability of the outcome.

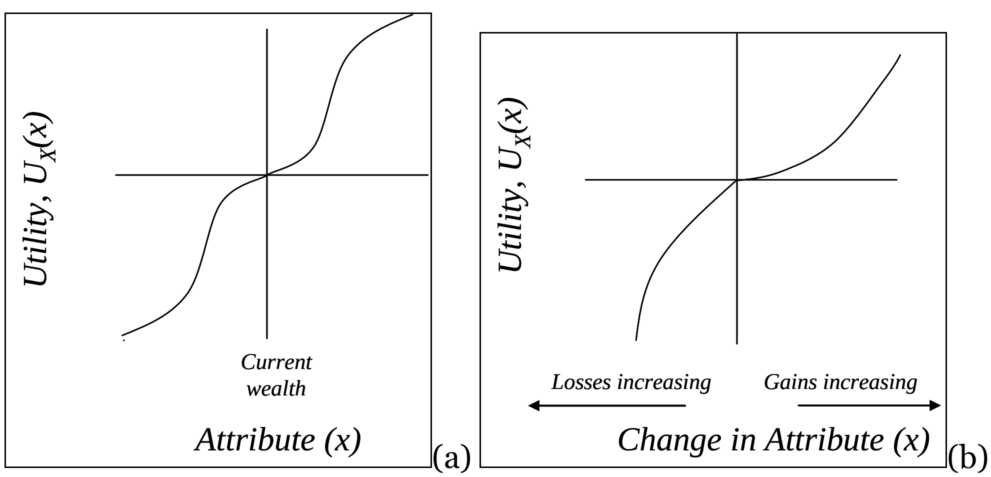


Figure 6. A Markowitz utility function (a) and the phenomena described by Kahneman and Tversky's prospect theory (b).

Value functions and utility functions appear similar, but are distinctly different. The value function describes a decision maker's attitude toward wealth and the utility function describes a decision maker's attitude toward *both wealth and risk*. The difference between a utility function and

[1] Some authors emphasize that Kahneman and Tversky's function is not actually a utility function, but rather a value function (Fischer 2004).

a value function is apparent in how the decision maker's preferences are assessed (Fischer 2004). The value function can be obtained by asking the decision maker a series of questions of the form: "For what amount of x would you be indifferent between a dollar amount y and that amount?" The utility function can be obtained by asking the decision maker a series of questions of the form: "What amount x would be equivalent to having either a dollar amount y with probability p or having nothing with probability $1-p$?" Value and utility functions are typically parameterized through the implementation of computer-based surveys designed to assist decision makers to assess their preferences. A discussion of these preference assessment techniques is beyond the scope of this report.

All utility functions are value functions, but not all value functions are utility functions. All utility functions can be defined as a strictly increasing monotonic function of the attribute level or the value function: $U(x) = f(V(x))$ (Keeney and von Winterfeldt 2007). Therefore, a utility function is equivalent to a value function when the decision maker is risk neutral. Value functions and utility functions may have similar functional forms and several useful forms exist. A common and convenient form of the utility function is the exponential utility function (Garvey 2008). The exponential utility function for an economic good can be written as follows:

$$U(x)=\begin{cases}\dfrac{1-e^{-(x-x_{MIN})/\rho}}{1-e^{-(x_{MAX}-x_{MIN})/\rho}} & ,\text{for } \rho \neq 0\\[2ex] \dfrac{x-x_{MIN}}{x_{MAX}-x_{MIN}} & ,\text{for } \rho = 0\end{cases} \tag{1}$$

The exponential utility function for an economic bad can be written as follows:

$$U(x)=\begin{cases}\dfrac{1-e^{-(x_{MAX}-x)/\rho}}{1-e^{-(x_{MAX}-x_{MIN})/\rho}} & ,\text{for } \rho \neq 0\\[2ex] \dfrac{x_{MAX}-x}{x_{MAX}-x_{MIN}} & ,\text{for } \rho = 0\end{cases} \tag{2}$$

An implicit assumption in this form of the exponential utility function is that the decision maker expresses a constant risk attitude over all levels of wealth. This is in contrast to the utility functions proposed by Markowitz (1952) and Kahneman and Tversky (1979). In the exponential form of the

utility function, ρ is called the **risk tolerance** parameter. This parameter determines the shape of the curve and reflects the combined effects of an individual's attitudes toward wealth and risk. If ρ is positive, the utility function is risk averse. If $\rho = \infty$, the utility function is risk neutral (the symbol ∞ means infinity, which can be approximated by a high number relative to the maximum possible amount of the attribute x_{MAX}). Utility functions with higher values of ρ describe the risk attitudes of individuals who have higher levels of risk tolerance. If ρ is negative, the utility function is risk seeking.

Figure 7 illustrates the exponential form of the utility function for three different levels of risk tolerance. All three utility functions characterize a risk averse individual, but a high value for ρ relative to maximum possible level of the attribute or wealth yields a utility function that approximates a risk neutral risk attitude (e.g., in Figure 7, this is illustrated for $\rho = 1x10^6$). The risk tolerance parameter has an intuitive meaning. Consider the decision to accept or forego a lottery as illustrated in Figure 8. This lottery, A_2, has an entrance fee ($x/2$) that is half the potential payoff amount x. The risk tolerance parameter of the exponential utility function is approximately equal to the largest value of x for which the decision maker would be willing to accept the lottery (Clemens 1996).

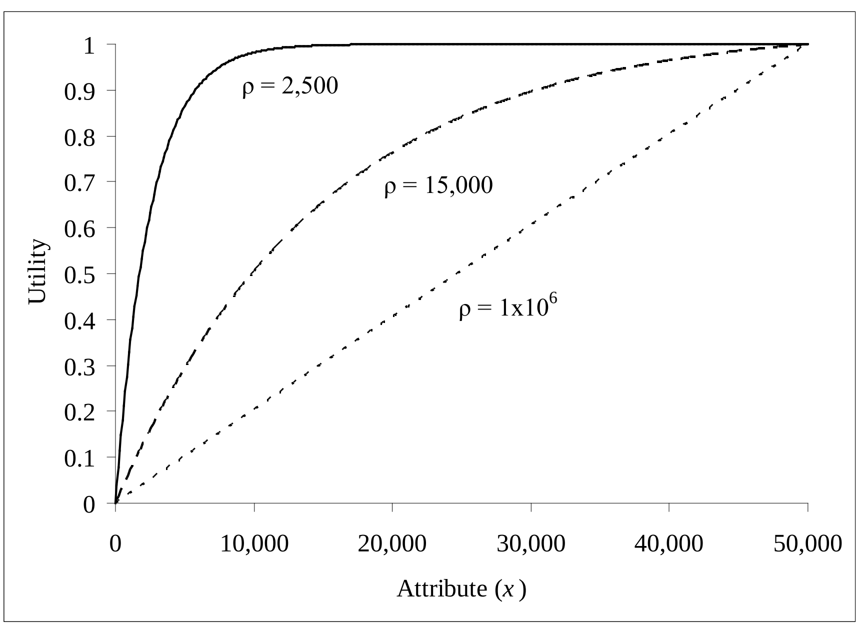


Figure 7. An exponential utility function for three possible values of the risk tolerance parameter, ρ. The values of the attribute may range from 0 to 50,000.

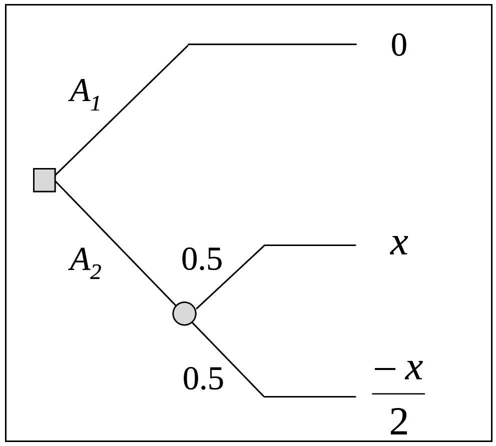


Figure 8. A choice between a sure-thing alternative, A_1, which will pay off 0, and a lottery, A_2, which has a 50% chance of paying off *x* and a 50% chance of costing –x/2. The risk tolerance is approximately the maximum amount *x* that would induce an individual to accept the lottery, A_2.

What risk attitudes should federal agencies adopt?

A utility function describes the wealth and risk attitudes of an individual. The theory leaves unanswered many questions about how groups should make risky decisions. A group is a set of two or more individuals with a stake in the outcome of the decision. For federal agencies, the stakeholder group includes all taxpayers and individuals who are potentially affected by a project. At present, there is no widely accepted theory that explains how groups should make decisions under uncertainty or how the individual utility functions of group members can be aggregated. Both are active areas of study in the field of social choice within the discipline of economics.

How should decision makers in federal agencies confront this problem when analyzing decisions under uncertainty? The "decision engineering" approach suggested by Ron Howard is to analyze the decision problem as if the group were an individual with its own utility function that is chosen by the decision maker (Howard 2007). For a private entity, that utility function should represent corporate goals and the acceptable risk levels for the corporation (Clemens 1996). Howard (1988) suggests guidelines for determining a corporation's risk tolerance in terms of total sales, net income, or equity (Clemens 1996, p. 480). Determining an appropriate risk tolerance may be more difficult for a public entity that is working on behalf of society because the stakeholders have a much more complicated set of interests and utilities that cannot be easily aggregated.

Howard's approach is a practical one, but is not without its problems for federal agencies. Decision makers in federal agencies may tend to exhibit a relatively high level of risk aversion. This high level of risk aversion might be attributed to an interest in preserving an agency's public image or the internal punishment and reward systems within the agency. Risky decisions put an agency's image at stake. When bad outcomes occur, the public is often quick to point the finger at the first public agency that can be blamed. In anticipation of this, agency leaders may understandably adopt a risk-averse attitude. Reward systems within an agency may also encourage risk aversion. For example, this might occur when decision makers are rewarded for good outcomes and punished for bad outcomes, as opposed to being rewarded for making good *decisions* and punished for bad *decisions*, regardless of the outcome.

A principal-agent problem exists if decision makers within an agency adopt risk attitudes that are more risk averse than those in the society for whom they work. A principal-agent problem is one in which the agent (the agency) who acts on behalf of the principal (society or taxpayers) has incentives that are at odds with those of the principal. If this occurs, the economic benefits realized from public investment will be sub-optimal and the opportunity costs of these decisions over the long term could conceivably be quite substantial. The choice of what risk attitudes an agency should adopt needs to be addressed at a policy level and is beyond the scope of this report. In the absence of any top-down guidance on risk attitudes, it should be understood that there are incentive structures within agencies that may lead decision makers to adopt risk-averse attitudes that lead to sub-optimal outcomes.

Analysts who are studying risky decisions will typically not have the information they need to specify a utility function. Therefore, this report recommends that analysts assume a risk-neutral utility function and conduct sensitivity analysis on the risk tolerance parameter to simulate increasing risk aversion. The question to be addressed through sensitivity analysis is whether or not and at what point changing the risk tolerance parameter changes the decision. Presenting the decision maker with information about what is the optimal solution to a problem for a range of risk attitudes enables the decision maker to retain control and responsibility for the decision. This approach is difficult when analysts and decision makers undertake studies with the objective of "getting the answer," but is a natural outcome when studies are undertaken with the objective of developing a comprehensive understanding of the decision problem.

4 Decision Analysis: A Practical Approach to Modeling Decisions Under Uncertainty

A decision model is a computational tool designed to assist a decision maker in evaluating and exploring the outcomes of the various decision alternatives in light of individual attitudes about wealth and risk and subjective assessments of uncertainty. Like all models, decision models are simplifications of reality and the ability to develop useful decision models requires practice. This chapter outlines the various steps of decision modeling using decision trees. Other methods such as influence diagrams exist and have advantages, but decision trees are easier to understand and there is a one-to-one linkage between the two approaches. To apply decision analysis, analysts and decision makers need to know the alternatives, the performance measures, the relevant sources of uncertainty in quantifying those performance measures, and the characteristics of those uncertainties.

Choose the decision frame

The first step in decision modeling is to choose the decision frame. This is done by identifying the alternatives and performance objectives. All of the potential alternatives exist in the **decision space**, an *n*-dimensional space defined over the decision variables, which are the set of discrete or continuous variables that make each alternative unique. Decision problems that have a large number of alternatives can be very difficult to solve; therefore, it is customary to sample a manageable set of alternatives from the decision space. There are at least two guidelines for selecting alternatives. First, a no-action alternative (maintenance of the status quo) should always be considered. Second, it is best to sample alternatives from all regions of the decision space. However, if it is known that a particular alternative or region of the decision space is not feasible, these alternatives can be explicitly excluded from further consideration. Such exclusions should be noted in documentation.

There may be external factors that limit the region of the decision space from which a set of alternatives can be sampled. For example, when considering flood damage reduction alternatives, the decision maker may only have the authority to implement structural damage reduction measures such as levees, floodwalls, or gates. In that case, any

consideration of non-structural alternatives such as population relocation might be moot. In general, one might tend to consider a wide variety of alternatives at higher levels of the organization and a more narrow set of alternatives at lower levels.

The decision maker must have a clear understanding of how the outcome of each alternative will be evaluated. While practitioners are often unclear about what performance measures should be used, there can be no lack of clarity about what performance measure will be used. The choice of a performance measure often reflects the values and priorities of the decision maker. Performance measures must be identical for each alternative and it is best to select performance measures early in the decision process. Performance measures can be refined and updated as the decision maker learns more about the decision problem, but this may require revisions to modeling efforts that support the decision-making process.

The choice of performance objectives will affect the choice of models used to simulate decision outcomes. For the purpose of example, assume that a decision outcome can be evaluated in terms of a single monetizable attribute. [1] Outcome performance must be forecasted over the life of the project with each alternative in place. The measure of outcome performance is the net present value (*NPV*) of the economic returns and costs over the life of the project. NPV_j is the present value of the future stream of economic returns X with the j^{th} alternative A in place minus the future stream of performance costs C aggregated and discounted over a planning horizon consisting of T intervals, t={1,2,...,T}:

$$\left(NPV \mid A_j, \mathrm{Y}, \mathrm{Z}\right) = \sum_T \frac{\left(X_t \mid A_j, \mathrm{Y}\right) - \left(C_t \mid A_j, \mathrm{Z}\right)}{\left(1+r\right)^t} \tag{3}$$

Economic returns and costs are functions of models (g, h) that contain a vector of uncertain inputs that are random variables, Y and Z: $X_t = g\left(\mathrm{Y}\right)_t$ and $C_t = h\left(\mathrm{Z}\right)_t$. Some inputs may be outputs of other models. Value parameters that may be incorporated into the various models, such as multiattribute weights and the discount rates, are not considered

[1] This report does not address the case of non-monetizable performance measures.

uncertain variables because it is assumed that the decision maker knows his or her preferences.

Identify and screen sources of uncertainty

Decision analysis enables the decision maker to address input and model uncertainties in forecasting economic returns and costs. The first task is to identify the important sources of input uncertainty in those calculations. This task should generally begin by considering the sensitivity of NPV to inputs of the decision model and, as sensitivities may warrant, working back to the inputs of any supporting models. Morgan and Henrion (1990) describe a useful measure of uncertainty importance η that is the product of the sensitivity of the model output (in this case NPV) and the standard deviation of an uncertainty distribution for the NPV model input, y:

$$\eta_y = \frac{\partial NPV}{\partial y}\sigma_y \tag{4}$$

Calculation of η requires the analyst or the decision maker to describe uncertainty in the input variable of the decision model. Uncertainties should be characterized using probability distributions. These can be objective distributions based on data or, if data are scant, informed subjective assessments of uncertainty. An appropriate statistical distribution function is then chosen to represent uncertainty in the input (a discussion of available distributions and their parameterization is covered in statistical texts). When the inputs of the decision model are the outputs of supporting models, the uncertainty distribution in the input variable should be propagated from input uncertainties in the supporting models.

Candidate uncertain variables are then ranked by the absolute value of uncertainty importance. Variables with the highest uncertainty importance are selected for further analysis. This is often done by looking for order of magnitude differences in uncertainty importance and then excluding those variables with relatively low uncertainty importance. As a practical matter, it is important not to consider too many uncertainties. Decision problems become exponentially more difficult to analyze as the number of uncertainties increases. As a general rule, it is best to start with a short list, analyze the decision problem, and then if necessary expand the number of uncertain inputs as the analysis may warrant.

Construct a decision tree

Decisions under uncertainty can be solved using a decision tree, which is illustrated in Figure 9. Figuratively, the decision tree consists of a tree-like sequence of nodes and branches leading to possible outcomes. Working from left to right in the figure, the first node in the tree is a decision node, which is customarily represented by a square. One branch emanates from the decision node for each of $j = \{1,2...,J\}$ alternatives. Each branch terminates in a chance node, represented by a circle. In Figure 9, two decision alternatives (A_1 and A_2) and one chance node (Y) are replicated for each decision. The chance node represents a source of uncertainty in predicting decision outcomes. That source of uncertainty can be captured by either a quantitative or a qualitative variable.

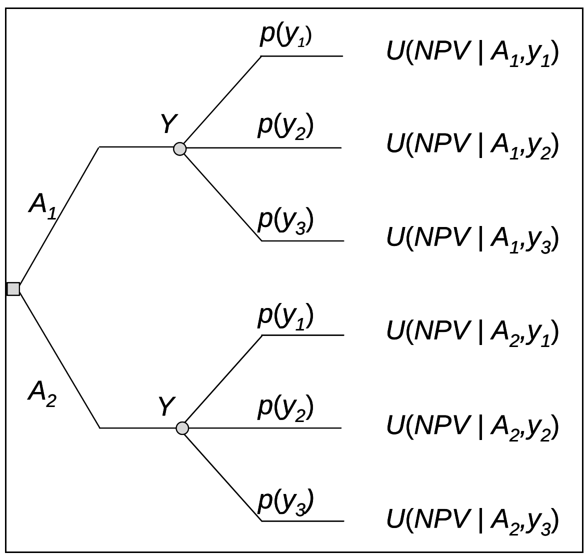


Figure 9: A decision tree with two alternatives and one source of uncertainty that has been discretized into three possible states. Outcomes are expressed in terms of the utility of the NPV of the outcome.

In Figure 9, the uncertain variable Y may take one of three discrete states, $Y \in \{y_1, y_2, y_3\}$. If Y is a qualitative variable, a label is used to describe the state. If Y is a quantitative variable, a number is used to describe the state. If Y is a continuous quantitative variable, decision analysis forces the analyst to discretize that variable. For example, continuous uncertain variables such as population and employment growth rates, sedimentation rates, and rates of sea-level rise may be discretized into a mutually exclusive and collectively exhaustive set that describes - for the purpose of decision modeling - all of the possible “states of the world” with respect to

that variable. While the discussion and examples in this report emphasize discretization of uncertain variable distributions, it is also possible to obtain quasi-exact solutions using numerical simulation methods that sample many values directly from the continuous distributions underlying the discrete probabilities. This report emphasizes decision modeling using decision trees rather than simulation to help make the discussion more accessible, not because decision trees are better than simulation. Simulation may tend to be preferred given access to modern software and computing tools.

The three possible states in Figure 9 describe the range of conditions under which the outcome of the decision may be realized. Each possible state is associated with a probability $p(y_k)$ that reflects the decision maker's beliefs regarding the conditions under which the outcome of the decision will be realized. The assignment of probabilities is discussed in the following section. If two or more uncertain variables are considered in a decision analysis, the probabilities of the states for one variable may be conditional on the state of the other variable, but the probabilities at each chance node must conform to the probability axioms. There can be any number of possible states, but a desire to keep the problem manageable often suggests that a limited number of possible states should be considered. What is manageable depends upon the sophistication of the computational tools with which one approaches the problem.

Decision making under uncertainty implies that the decision maker knows something about his attitudes toward wealth and risk. Therefore, the outcomes of the decision are expressed in the form of utility scores $U(NPV)$. The expected utility of an alternative $E[U(NPV \mid A_j, y_k)]$ is the expected utility of the NPV of the outcomes. This is calculated by converting the NPV of each decision outcome to a utility score and calculating a probability weighted sum of the utility scores for the alternative:

$$E[U(NPV \mid A_j, y_k)] = \sum_k p(y_k) U(NPV \mid A_j, y_k) \tag{5}$$

A decision analysis is "solved" by identifying the alternative that maximizes the expected utility of the decision maker. Many applications of decision analysis neglect the utility calculation. This is synonymous with assuming that the decision maker is both risk neutral and wealth neutral. This may be a justifiable assumption if the stakes are small in relation to

wealth, but it should be an explicit assumption of the analysis. If the utility function is neglected, decision analysis only leads to a rational decision if the decision maker really is neutral with respect to wealth and risk.

Probabilities for uncertain variable states

Each branch emanating from a chance node represents a possible state of the world and must be associated with a probability. That probability represents the decision maker's assessment that the conditions under which an outcome will be realized are accurately described by that state and not the other possible states. While these probabilities may be determined either subjectively or objectively, it is useful to consider in somewhat more detail the origin of these probabilities. The branches emanating from each chance node are propositions. In a properly formulated decision tree, the possible states are collectively exhaustive and mutually exclusive; therefore, exactly one of the possible states must be true. The probabilities in a decision tree represent the decision maker's subjective assessment that the proposition $Y = y_k$ is true.

The Bayesian underpinnings of subjective expected utility have already been discussed. Bayesian probabilities represent degrees of belief in a proposition rather than an objective frequency. Because probabilities represent the views of the decision maker, it is always possible to assign probabilities consistent with those views whether or not those views are based on objective information. If the decision model is properly structured, the decision maker maximizes subjective expected utility using those probabilities because they are consistent with his beliefs. Thus, decision makers can make decisions in the face of uncertainty using the subjective expected utility model given what they know, understand, or believe. This has great value, even in a science-based organization that relishes making decisions based on facts. As decision makers know too well, decisions must often be based on partial information, so-called facts may be imperfectly known, and stakeholders may be in disagreement about the facts.

The foregoing discussion is not an argument for making decisions prematurely or in the absence of facts. The best decisions will be well-informed by facts. Rather, the foregoing discussion implies that the decision making under uncertainty methods described here can be successfully applied to maximize subjective expected utility even when little information is available. Therefore, the methods can be scaled

appropriately for use in many situations, including those where there may be very limited information.

What are the ways that a decision maker can assign probabilities to possible states? The available options can be arrayed in terms of how much quantitative information the decision maker has available to formulate his beliefs and how closely he aligns his beliefs with the quantitative information that is available. This concept is illustrated in Figure 10. The horizontal dimension indicates what type of information is available. The vertical dimension indicates how closely the decision maker aligns his beliefs with the available quantitative information. Within this array, there are a wide range of possible approaches. For expedience, this report emphasizes two extreme approaches. These extremes are the nominal assignment of probabilities and derived probability assignments.

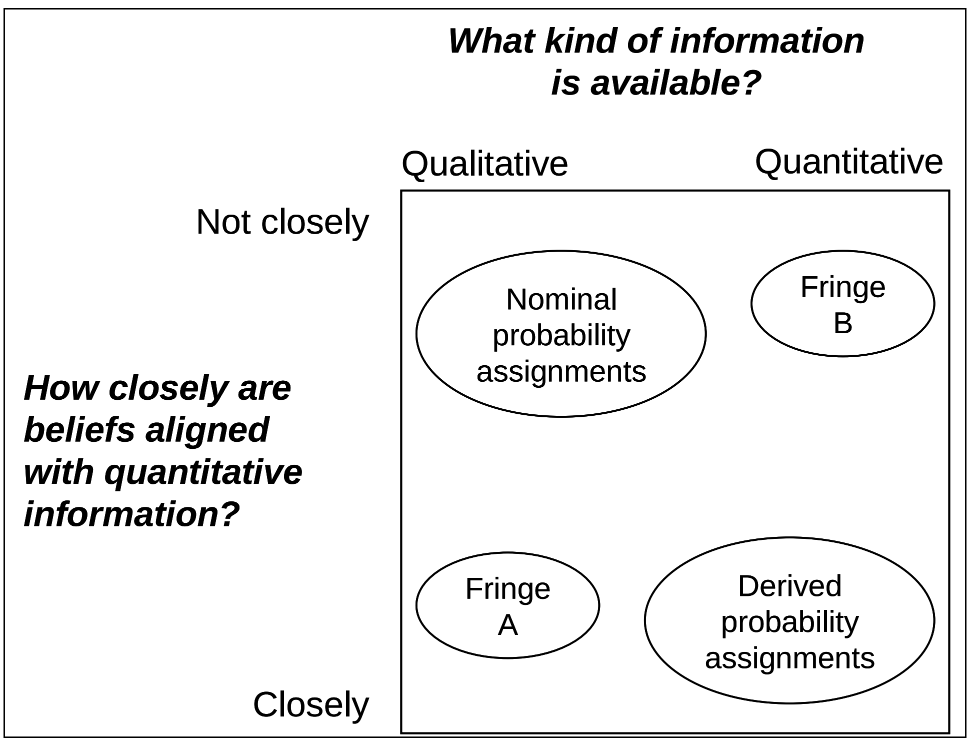


Figure 10. Options for assigning probabilities to uncertain variable states.

The nominal approach is simply to have a decision maker state his belief with regard to the probability that the proposition is true given what is known to him at the time. This approach emphasizes the subjective nature of decision analysis and can be used in situations where little quantitative information is available or the decision maker chooses not to rely on the quantitative information. This is also appropriate when the uncertain

variables are more qualitative. For example, in describing the weather, a decision maker may be more concerned with whether or not it rains tomorrow, in which case there are two possible states associated with the chance node (rain and no rain), but there are no underlying quantitative scales describing the amount, intensity, or duration of rainfall that might occur. The nominal approach is appropriate if the information available to the decision maker is mostly qualitative. Some decision makers may be reluctant to acknowledge the existence of any beliefs in the absence of quantitative information. However, the existence of these subjective probabilities can be proven by teasing them out through a series of laboratory experiments. Therefore, if decision makers are reluctant to assign nominal probabilities, a middle way may be to extract their beliefs through a series of experiments. For decision makers who may be operating outside their domain of expertise, it is also sometimes possible to obtain probabilities from experts. When relying on expert opinion, it is important to use formal structured techniques to elicit those probabilities.

At the other extreme, it is possible to have mostly quantitative information available and to pay close attention to that information while de-emphasizing qualitative information that might be available. When there is sufficient information, probabilities for each state can be derived from probability distributions. For continuous variables, this is accomplished by breaking the variable's range into intervals and obtaining a probability mass for each interval. An example is illustrated in Figure 11. For example, suppose that a continuous uncertain variable y is characterized by a density function $f_Y(y)$ and the uncertain variable will be discretized to three possible states: "Low" with a nominal value of 5, "Medium" with a nominal value of 15, and "High" with a nominal value of 25. The probability of each state can be calculated by constructing intervals centered on the chosen states of Y: $p(y_k) = F_Y(y_k^{MAX}) - F_Y(y_k^{MIN})$, where $F_Y(y) = \int_{-\infty}^{\infty} f_Y(y)dy$ and the variables y_k^{MIN} and y_k^{MAX} are the lower and upper bounds of the k^{th} interval. The probability of being in each state in Figure 11(b) is equal to the area under the density function in 11(a). Discretization has advantages over nominal assignments of probability because it ensures a coherent probability structure, facilitates updating of probabilities for adaptive management decisions, and facilitates sensitivity analysis.

It seems likely that most decision makers will operate somewhere on the diagonal of the array in Figure 10, between (Qualitative, Not closely) and

(Quantitative, Closely). Decision makers who operate in the areas labeled Fringe A and Fringe B may be ignoring available information. In the region of Fringe A, the decision maker has mostly qualitative information available to inform subjective probability assessments. In such cases, it would seem unreasonable to emphasize quantitative information. The opposite is true for the region denoted as Fringe B. If most of the information that is available is quantitative, it seems that a decision maker should focus on that information rather than qualitative information. The point being made here is that decision makers will usually need to integrate quantitative and qualitative information when analyzing decisions under uncertainty.

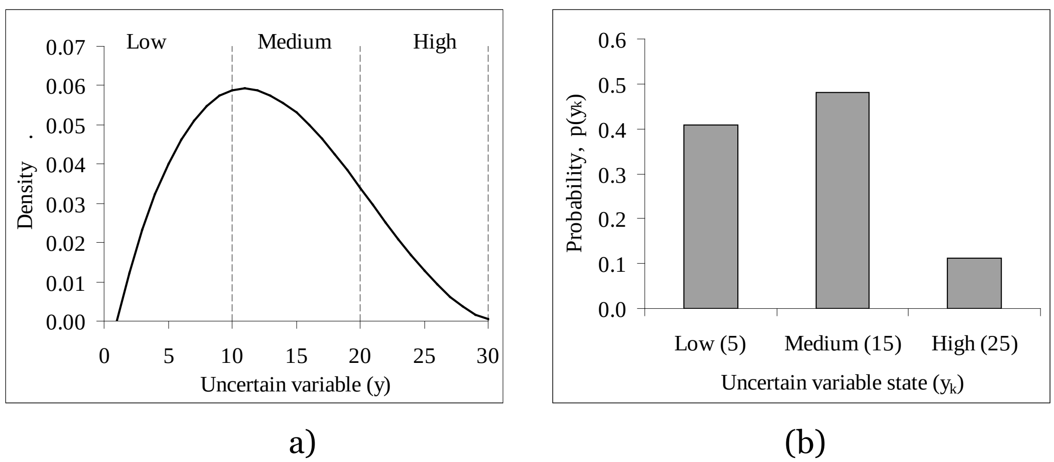


Figure 11. Density function for a continuous variable (a) is discretized to obtain the probability of being in one of three nominal states (b).

Assess outcomes for each scenario and calculate their probabilities

A scenario is a set of states for one or more uncertain variables. Scenarios describe the conditions for which the decision outcomes are evaluated. For a set of n uncertain variables, each with k states, there are k^n scenarios, assuming all the possible combinations of variable states are feasible in reality. For example, a decision problem with two uncertain variables, Y and Z, each with three states, has $3^2 = 9$ possible scenarios. These scenarios represent all possible states of the system in the decision model. In the real world many variables may be continuous or may naturally have many more levels than are represented in the model. Each scenario is associated with a probability of occurrence that is conditional on the state of other uncertain variables. A coherent set of scenarios is a set of scenarios that encompasses all possible states of the system such that every possible state in the real world can be associated with exactly one

scenario. The sum of probabilities for each of the scenarios must equal one. Outcomes are assessed for each scenario by evaluating the outcome of the decision under the conditions described in each feasible scenario.

Characterize uncertainty in the outcome of each alternative

It is often useful to plot the cumulative probability distribution function over the decision outcomes. Such plots are sometimes known as risk profiles because they show how the probability is distributed over the potential outcomes. The cumulative distribution for an outcome is constructed by ranking the consequences for an alternative in order of increasing consequence and calculating the cumulative probability of realizing each of the possible outcomes. The result can be plotted as a step function as illustrated in Figure 12. Figure 12(a) shows less uncertainty in the outcome of Alternative 2 (A_2) than Alternative 1 (A_1). In this example, A_1 has a higher expected *NPV*, but a risk-averse decision maker might still prefer A_2 if it has a higher expected utility than A_1. Figure 12(b) illustrates a case where the *NPV* outcomes of A_2 are always better than the NPV outcomes of A_1. In this case, A_2 exhibits stochastic dominance because the outcomes under this alternative are always better than under A_1.

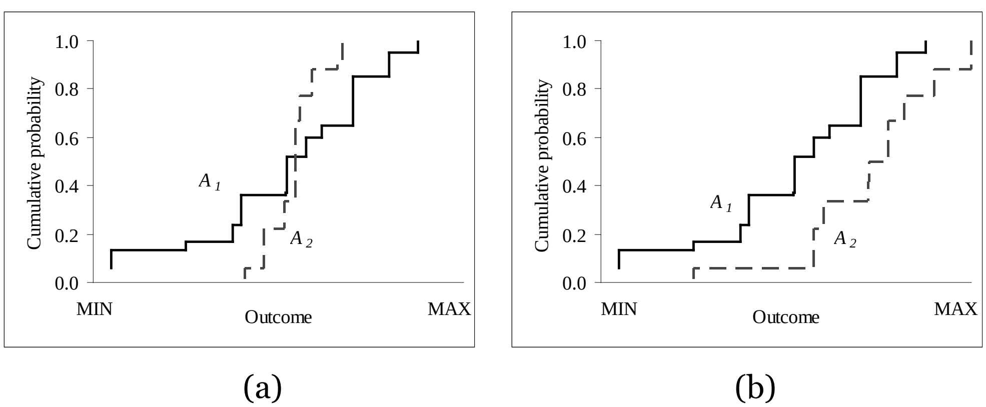


Figure 12. Examples of risk profiles for two hypothetical alternatives.

Risk profiles should not be confused with risk curves introduced in Figure 2, but they are closely related to each other. Risk curves are loss-exceedance curves that give the probability of exceeding a potential loss; thus, they are only evaluated over adverse outcomes. In contrast, risk profiles are evaluated over all potential outcomes. Risk curves give the probability that losses exceed a specified amount. In contrast, risk profiles give the probability that an outcome is less than a specified amount.

Calculate utility scores and maximize expected utility

The decision problem is solved by finding the optimal alternative. An alternative is optimal if it maximizes expected utility. Utility scores can be calculated for each outcome using the exponential utility function described in Chapter 3 (section titled "What is a utility function?"). The minimum utility score is 0, for the worst possible outcome, and 1 for the best possible outcome. A calculation of utility scores requires an assessment of the decision maker's risk tolerance, which can be accomplished through a series of experiments. A discussion of these experiments is beyond the scope of this report. The value of the utility function is that it incorporates information about the decision maker's preferences regarding risk and wealth. A ranking of the outcomes based on utility will be the same as a ranking based on the outcomes themselves, but the strength of preference for each outcome is adjusted based on the decision maker's risk tolerance. The expected outcome and the expected utility are calculated by weighting the outcome or the utility scores by their probabilities and adding them up. At this point, it is important to point out that the expected utility is not the same as the utility of the expected outcome and decisions that are made by maximizing expected utility may differ from decisions that are made by maximizing expected outcomes. The two methods lead to the same decision if a decision maker is risk-neutral.

Analyze sensitivity of the decision model

The results of a decision analysis depend upon the parameter values and probability distributions that are used in modeling the decision. As with all models, it is useful to assess the sensitivity of modeling results. The objective of a sensitivity analysis is to test the conclusions of the analysis and evaluate the importance of key parameters and assumptions in the decision model. The question driving the sensitivity analysis should be "Would changes in parameters or other assumptions in the decision model alter the potential outcomes of the alternatives or change the optimal decision?" Sensitivity analysis enables the analyst or decision maker to pick the problem up, examine it from a variety of perspectives, and play with it to challenge its conclusions. The process breeds familiarity with the decision problem and the decision model and builds confidence in the results of the analysis.

Sensitivity analysis shows the decision maker what impact key assumptions in the decision model might have on the net present value and

utilities of each alternative and the optimality of each alternative. Sensitivity analyses are developed by solving the decision model systematically varying probability and parameter assumptions. Parameter assumptions to consider might include the discount rate, risk tolerance parameter, weights on multi-attribute value functions, and variables that are considered uncertain, but have not been included as chance nodes. A decision model may contain many more relationships than can be effectively considered by a decision maker. In this case, the analyst should emphasize the pivotal relationships when presenting sensitivities to decision makers. The pivotal relationships are those that might change the decision.

When conducting a sensitivity analysis, it is critical to understand the context in which the decision is being made. If the decision maker is an individual, the sensitivity analysis should focus on fixed variables and parameters in the decision model because value parameters and subjective probability assessments should be known to the decision maker. However, if the decision analysis is being prepared for a third party whose values and subjective probability assessments are not well known to the analyst, it is also useful to show the sensitivity of the decision to value parameters and parameters of subjective probability distributions. This lets the decision maker find the optimal alternative based on his own values or beliefs. A good sensitivity analysis makes information available and lets the decision maker retain control over the actual decision.

In the context of a collaborative decision-making process, sensitivity analysis can be an important tool for resolving differences among stakeholders and developing consensus. The sensitivity analysis should help stakeholders to understand how different points of view might lead to different courses of action. Therefore, an analysis of sensitivity with regard to value parameters and subjective probability assessments is appropriate. If differing points of view would not lead different decision makers to select different courses of action, then the differences can be set aside. This provides a way of focusing deliberation on the relevant issues and setting aside irrelevant differences.

It can be easy for an analyst to become overwhelmed with the task of considering all possible sensitivities and to overwhelm the decision maker with confusing or irrelevant information. This should be avoided by focusing on only those relationships that, after careful consideration of the

assumptions and uncertainties in the context of the particular decision being made, are deemed most critical to the decision maker.

The value of including uncertainty in the decision model

In general, one should consider the uncertainty in a decision when the uncertain variables could affect the net benefits of a decision and there is a potential loss or opportunity cost associated with choosing an alternative that leads to a suboptimal outcome. However, there can be substantial costs associated with conducting a detailed probabilistic risk and decision analysis. Investments in a risk and decision analysis should be scaled to reflect the potential benefits of an analysis. The benefits of an analysis are the avoided losses that are expected from an analysis. The expected value of considering an uncertainty in a decision analysis is the difference between the expected outcome of the alternative one would choose considering that uncertainty and the expected outcome one would choose ignoring that uncertainty. The risk premium is an upper bound on including uncertainty in the decision process (Morgan and Henrion 1990). Because calculating the risk premium requires a probabilistic analysis, an estimate of the risk premium is often based on initial screening-level analyses undertaken using lower levels of resolution, "back of the envelope" calculations, and available information. The value of including uncertainty in decision analyses is discussed by Morgan and Henrion (1990) and by Lund (2008).

The value of perfect information

Information that reduces uncertainty has value only if it would alter the decision maker's decision about which alternative is preferred. If there are costs associated with obtaining information to reduce uncertainty, the optimal amount of information in a decision process is rarely perfect information. Perfect information means that there is no uncertainty with regard to the models or the quantities or assumptions used in the models. Sometimes it is useful to calculate the expected value of perfect information (EVPI) as a way to assess whether or not investments to resolve those uncertainties are likely to pay off. If the expected costs of obtaining the information exceed the expected benefits of having that information, then one is better off making the decision despite the uncertainty.

The expected value of perfect information (EVPI) about an uncertain input in a decision analysis is the difference in the expected outcome of a deci-

sion that is made with perfect information and the expected outcome of the same decision made without that information. For a risk-neutral decision maker and a decision with a single uncertain variable, EVPI is calculated as follows:

$$EVPI = \sum_k \left[\underset{j}{MAX} \left[NPV \mid A_j, y_k \right] \right] p(y_k) - \underset{j}{MAX} \left[\sum_k \left(NPV \mid A_j, y_k \right) p(y_k) \right] \tag{6}$$

All variables are as previously defined in this report. If there are multiple sources of uncertainty in a decision, say for example variables *y* and *z*, then a partial EVPI (EVPYI) - in this case for the uncertain variable *y* - can be calculated as follows:

$$EVPYI = \sum_m \sum_k \left[\underset{j}{MAX} \left[NPV \middle| A_j, y_k, z_m \right] \right] p(y_k) p(z_m) -$$

$$\underset{j}{MAX} \left[\sum_m \sum_k \left(NPV \middle| A_j, y_k, z_m \right) p(y_k) \right] p(z_m) \tag{7}$$

Equation 7 can be extended to account for as many variables as necessary. If the decision maker is risk averse, then the NPVs must be converted to utilities. Value of information analyses are discussed in detail by Yokota and Thompson (2004(a) and (2004(b)).

The EVPI statistic represents the maximum amount that a rational decision maker would be willing to pay to obtain perfect information before making the decision and depends upon the decision maker's initial subjective assessment of uncertainty regarding the inputs in question. Value of information analyses have an important place in decision analyses because they can be used to prioritize information-gathering needs and establish boundaries on how much to invest in resolving uncertainty.

It is rarely possible to obtain perfect information. Therefore, when prioritizing information-gathering needs using an EVPI calculation, one should consider the feasibility of obtaining perfect information. If perfect information cannot be obtained, as will be the case for many inputs to a decision model, one can also calculate the expected value of sample information (EVSI), which is sometimes called the expected value of imperfect information (EVII). EVSI requires the decision maker to characterize the uncertainty in the information that is being valued before making the decision.

5 Adaptive Management and Anticipatory Engineering

Adaptive management

Decision analysis provides a framework for the systematic implementation of adaptive management. Adaptive management is the process of adjusting recurring decisions over time using evidence obtained by monitoring. Recurring decisions are those that are made periodically on a continuing basis. The implementation of adaptive management involves several steps. The first step is to develop and apply decision analysis as described in this chapter. A monitoring program is established to collect new evidence about uncertain quantities in the decision model. Bayes' rule is then used to update the probabilities of the decision model in the next iteration of the decision. Bayes' rule gives the posterior probability $p(y_k|e)$ which is the probability of uncertain variable y being in the k^{th} state given new evidence e:

$$p(y_k \mid e) = \frac{p(y_k)p(e \mid y_k)}{\sum_{k=1}^{K} p(y_k)p(e \mid y_k)} \tag{8}$$

The first term in the numerator is the prior probability of y being in the k^{th} state, $p(y_k)$. The second term in the numerator is the likelihood, which is the conditional probability of observing e given that y is in the k^{th} state. The denominator is the total probability of observing e under the prior probability distribution.

Framing one-shot decisions with potentially high costs as adaptive decision problems has the potential to mitigate the cost of "making the wrong decision," (*i.e.*, deciding one thing and then realizing that, in retrospect, after the future is revealed, we would have preferred to do the other thing after all.) The cost of implementing adaptive management is likely to be higher than the cost of implementing a one-shot decision, but for contentious decisions where contention originates from alternative subjective probability assessments, adaptive approaches have the potential to reduce political barriers to moving forward.

Two forms of adaptive management are recognized in the literature: passive and active. Passive adaptive management is accomplished using information that is collected opportunistically, without the benefit of control groups, randomization, or replication. Active adaptive management is accomplished using information that is collected through scientific experiments that are designed specifically for the purpose of adjusting the decision in an adaptive management process over time. While passive adaptive management is relatively inexpensive, the reliability of this approach has been questioned because of the lack of control over information collection (Prato 2005). If adaptive management will be used in a decision process, it is a good idea to work out in advance specifically how information will be collected and used.

Anticipatory engineering

Some decisions are more amenable to adaptive management than others. As adaptive management is described above, a recurring decision is needed to implement the procedure so that one has the opportunity to update probabilities based on new information. If a decision is non-recurring, there is no opportunity to update the decision. For example, one-time, irreversible, up-front decisions regarding construction projects may be difficult to update. These types of decisions are sometimes known as one-shot investment decisions. Once the decision to build has been made and ground is broken, it is very difficult to alter the design of these structures based on new information. However, this does not mean that adaptive management principles do not have meaning for construction decisions. Efforts to capitalize on adaptive management principles for these types of decisions have led to the concept of adaptive or anticipatory engineering.

Anticipatory engineering is the practice of building specific features into structures that enable managers to respond cost-effectively to changing infrastructure demands as the future unfolds. For example, sea-level rise may be expected to reduce the level of protection provided by a planned levee system. One possibility is for engineers to add additional levee height as a hedge against worst-case sea-level rise. Of course, the additional freeboard requires a significant up-front investment, the benefits of which may never be realized because the rates of sea-level rise are uncertain. If the worst case never occurs, the levee will have been over-built. This results in an opportunity loss because the money could have been spent to reduce flood risk elsewhere.

The adaptive engineering solution to the hedging problem is to build in the capacity to raise levee crowns should future sea-level rise occur, but not to raise the levee crowns until the uncertainty in the future rate of sea-level rise is reduced or revealed. Anticipatory engineering requires creative thinking about how the demands on infrastructure may change in the future and what kinds of anticipatory features might be built into infrastructure investments. However, the up-front costs of anticipatory engineering features must be justified by the discounted stream of future benefits, which is an expected cost savings.

Decision analysis can be used as a framework for evaluating adaptive engineering alternatives. Doing so requires expanding the set of alternatives to include both anticipatory and non-anticipatory designs and building the cost of "anticipatory" engineering features into the outcomes of alternatives. The outcomes of the decision analysis include the cost of building in the capacity to "upgrade" the infrastructure in the future, but not the cost of upgrading the structure should the conditions occur in the future. That is a separate decision to be made at a future time.

The decision to upgrade the structure in the future is a recurring decision that is suitable for the implementation of adaptive management techniques through which monitoring data are collected and the probabilities in the decision tree are updated each time the decision is reconsidered. For example, suppose that extra footprint has been purchased to facilitate future upgrades to a levee project and the project manager must re-evaluate the decision to upgrade the structure at five-year intervals. Monitoring data could be obtained during the five-year period leading up to reevaluation of the decision and used in conjunction with Bayes rule to update the probabilities over sea-level rise states in the decision model.

6 A Decision Analysis Example: Charter Fishing Boat Investment

This example demonstrates how the decision modeling techniques described in Chapter 4 could be applied to solve a relatively simple investment decision. The decision model is used to characterize the risks associated with a charter fishing boat investment. The investment risks arise from uncertainty in the costs of operating a charter fishing boat and the market for charter fishing boat services. The decision tree provides a functional model of the decision and facilitates an exploration of the sensitivities to probability assessments and value parameters, providing additional insights into the decision problem. The expected value of perfect information is calculated for each of the uncertain variables in the decision model to determine which sources of uncertainty should be resolved first and how much should be invested in that information. This example is also used to demonstrate how a decision model can be used to implement adaptive management by updating the probabilities in the decision model with new information using Bayes rule and reevaluating the investment decision.

The decision problem

Rick Barton, who is recently retired, has the opportunity to pursue his life-long dream of operating a charter fishing business on the Florida coast. Rick has located a used charter fishing boat that he can purchase for $52,500. He realizes that this is a risky investment with many uncertainties, but he must reach a decision on whether or not to buy this boat by the end of the week. If he does not operate a charter fishing business, he will leave the money in an existing investment that is guaranteed to yield an 8% annual return. Rick's decision has been framed as a choice between a charter boat investment and an alternative investment. A natural performance measure for this decision is profit. If the profits from his charter boat investment would exceed the returns from the alternative investment, he will invest in the charter boat.

Rick has acquired information on the operations of other charter fishing operations to assist him in making this decision. He develops a detailed cost and revenue model to estimate profit from operation of the fishing

boat. This model includes prices and amounts that are bound to fluctuate and cannot be known for certain (for example, fuel prices, fuel consumption, annual insurance rates, the cost of fishing tackle and bait, onboard accommodations, and advertising). Following a thorough analysis of the uncertainties in his cost model, Rick identifies three variables that he believes are most important in terms of how uncertain their values are and how much impact they may have on profit. They are the vessel operating costs, the booking rate, and the percent capacity. The booking rate is the fraction of days during the fishing season that the vessel is chartered. Capacity utilization is the fraction of vessel capacity that is actually used.

The decision model

A decision tree can be formulated to model the decision problem, as shown in Figure 13. At the decision node, there are two alternatives: a charter boat investment and an alternative investment. With an annual interest rate of 8%, the alternative investment leads to a certain outcome of $56,700 after one year. The outcome of the charter boat investment depends upon three uncertain variables, represented by chance nodes. Based on his analysis of other operations, Rick determines that his operating costs can vary widely, but will depend largely upon how hard it is to locate fish. In a good fishing year, the search time is low, resulting in lower fuel and maintenance costs. Operating costs can vary widely, but Rick determines that, for a good fishing year, a representative operating cost is $150/day and, for a bad fishing year, a representative operating cost is $225/day.

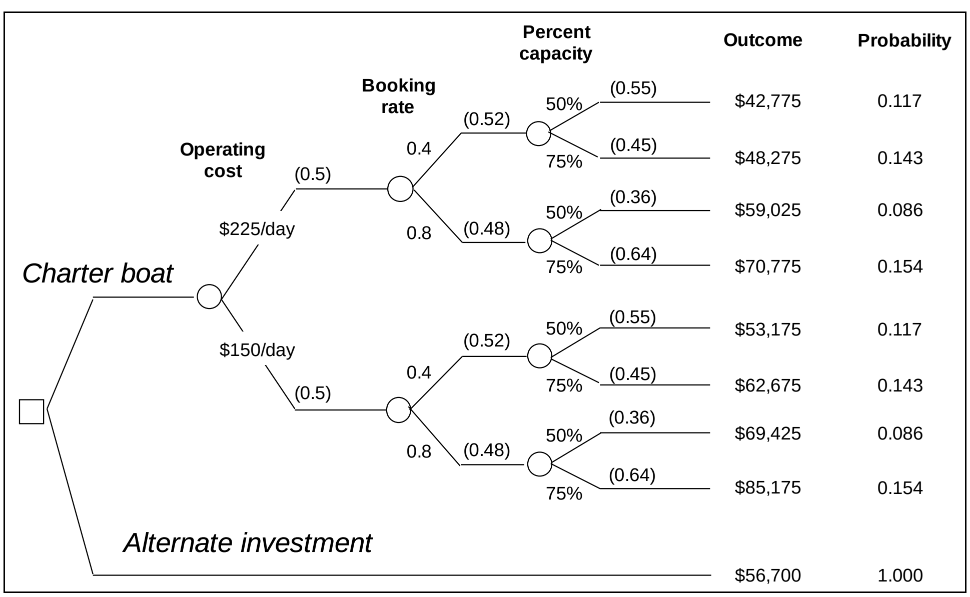


Figure 13. A decision tree for Rick Barton's charter fishing boat decision.

The other two variables of interest are the booking rate and capacity utilization. If the booking rate is low, the boat will sit in port many days and Rick will incur moorage, insurance, and other maintenance costs. If the booking rate is high, these costs will be offset by revenue. Based on his review of other operations, booking rates can range from 0.2 to 1.0. Rick chooses the mid-point of two equal intervals over the range of possible values to use in his decision analysis, so his possible values are 0.4 and 0.8. Capacity utilization is important because there is a per-person surcharge for fishing parties larger than two. The vessel has a capacity of eight fishermen, so the size of fishing parties will range from one to eight. Rick figures that, on the low end, average capacity utilization could be closer to 50 percent and, on the high end, average capacity utilization could be around 75 percent.

Probabilities are assigned to each possible state of the uncertain variables based on a review of other charter fishing boat operations. In Figure 13, probabilities appear in parentheses. Rick determines that good fishing years and bad fishing years occur at similar rates, so the probabilities of high and low operating costs are about the same (0.50). Oddly, the booking rate appears to be independent of whether or not it is a good fishing year or a bad fishing year. Rick analyzes his data and determines that booking rates are somewhat more likely to be on the lower end (0.52) than on the higher end (0.48). Capacity utilization varies with the booking rate. If booking rates are low, capacity utilization will also tend to be low and if booking rates are high, capacity utilization will tend to be high. Therefore, he calculates conditional probabilities for these chance nodes based on the booking rates. If booking rates are low, the probability of low capacity is 0.55 and the probability of high capacity is 0.45. If booking rates are high, the probability of low capacity is 0.36 and the probability of high capacity is 0.64.

The monetary values at the terminus of each branch of the decision tree in Figure 13 are decision outcomes. For example, the outcome of the alternative investment is $56,700, which is the $52,500 that was not invested in the boat plus $4,200 interest. For the investment in the charter boat, decision outcomes are calculated by setting the uncertain variables in the detailed cost and revenue model (operating cost, booking rate, and percent capacity) to simulate a scenario. Eight scenarios are evaluated and the potential outcomes range from $42,775 to $85,175. Some outcomes are worse than the alternate investment, so there is a risk of an opportunity loss. Each scenario is associated with a probability as indicated in the

column to the right. The probabilities for each scenario are calculated by multiplying the probabilities along each branch of the decision tree.

The decision

After careful study of other charter fishing boat operations and after an iterative process of model building, Rick is finally satisfied that his decision model provides a reasonable representation of the decision problem. However, he knows that he has more work to do before he can make a decision. Because his ability to recover from a financial loss during retirement will be limited, Rick understands that he is risk averse. Incorporating this information about his risk preferences into the decision analysis requires converting the potential outcomes into utility scores. However, Rick must first assess his risk tolerance. This he does with the help of a decision analyst friend. Based on Rick's responses to a series of questions about his attitudes toward accepting risk, the decision analyst friend estimates Rick's risk tolerance to be about 10,000. Armed with this estimate of his risk tolerance, Rick is able to convert his decision outcomes into utility scores, as shown in Figure 14. The utility scores are calculated using the exponential utility function as described in Chapter 3 of this report.

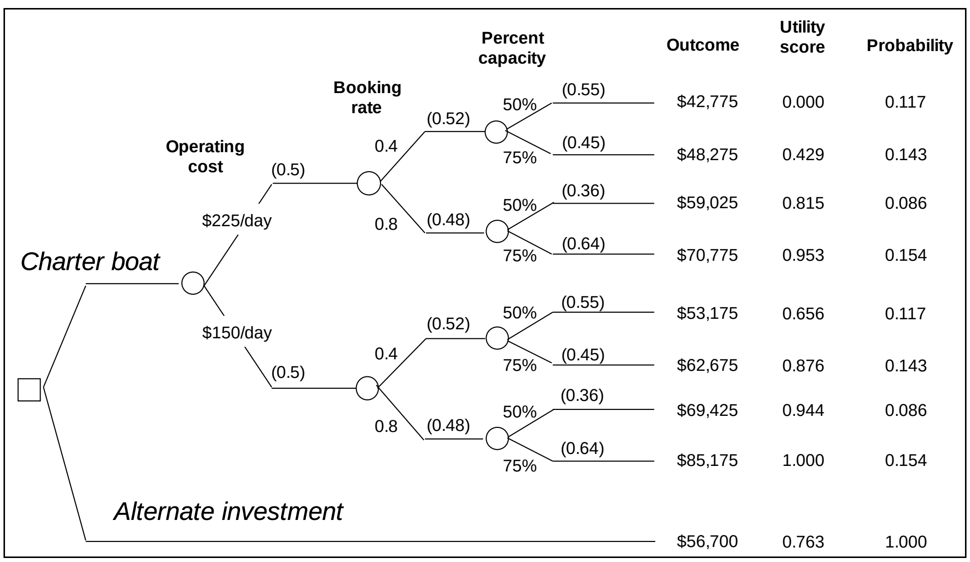


Figure 14. A decision tree showing utility scores for Rick Barton's charter fishing boat decision.

Rick calculates the expected utility of each decision alternative by weighting the utility scores by their probabilities and adding up them up for each alternative. The expected utility of the charter boat investment is 0.715 and the expected utility of the alternate investment is 0.763. Given

Rick's expressed preferences regarding risk, the best alternative for him is the alternate investment. A reevaluation of the charter boat investment decision might be warranted if economic conditions were to change. For example, a change in fuel prices or the price of the boat might warrant a reevaluation of the decision. Nevertheless, Rick feels somewhat disappointed in the outcome of his analysis. The expected utilities are very close together and he isn't really sure that he prefers the alternate investment. How can Rick be sure that he is really making the right decision by foregoing the charter boat investment opportunity?

Sensitivity analysis

Rick has been left feeling disappointed in the outcome of his decision. He consults his decision analyst friend who points out that discomfort with the results may signal shortcomings in the analysis. An obvious shortcoming in Rick's analysis is that his performance measure neglects important aspects of the decision outcome. For example, the psychic benefits Rick might derive from operating the charter fishing boat have not been considered. Unfortunately, there is no time to revise Rick's analysis by converting his performance measure to a multiattribute utility before the decision deadline. However, the friend also points out that Rick has not subjected the decision model to any kind of sensitivity analysis. Sensitivity analysis is an important step because it provides insights into the decision model and can reveal critical assumptions that may need to be revisited.

In a sensitivity analysis, the outcomes of a decision are explored over a range of input values. To introduce sensitivity analysis, first consider how a risk-neutral decision maker might approach the charter boat decision. Figure 15 (a) illustrates a sensitivity analysis that explores how the expected net present value of the decision changes in response to assumptions about probabilities for two uncertain variables. The x-axis is the probability that the booking rate is low (Y= 0.4) and the y-axis is the net present value of the decision outcome. The dashed line shows the expected net present value of the decision outcome for the alternative investment (A_2), $E[NPV|A_2]$ is constant at $56,700. The solid lines show the expected net present value of the decision for the charter boat investment (A_1), $E[NPV|A_1]$. If the solid line is below the dashed line, the alternative investment (A_2) is preferred. If the solid line is above the dashed line, the charter boat (A_1) is preferred. The three solid lines illustrate three realizations of the NPV function to further assess the sensitivity of the decision to operating cost assumptions. The nominal probability for the

operating cost being high is p(Z = \$225/day) = 0.5. If p(Z = \$225/day) is reduced to 0.1, then the charter boat is always preferred. If p(Z = \$225/day) is increased to 0.9, the charter boat is preferred if the probability that the booking rate is low, p(Y = 0.4), is less than 0.55.

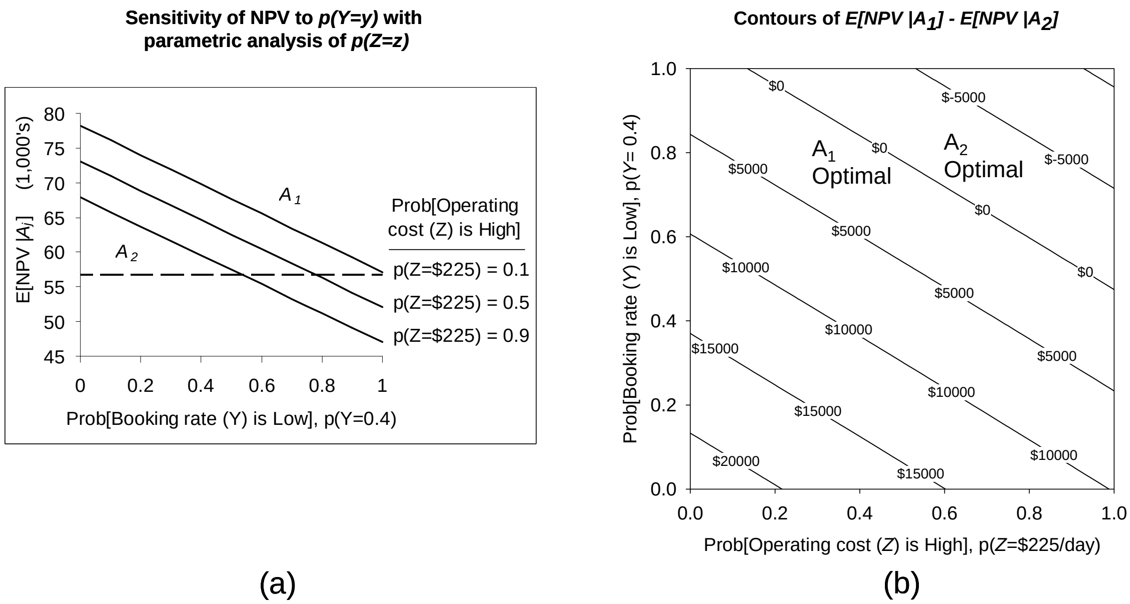


Figure 15. Sensitivity analysis showing expected NPV for the charter boat alternative (A_1) and the alternative investment (A_2).

A different view of the sensitivity analysis is offered by the contour plot in Figure 15 (b). The contour plot contains the same information as plot (a), but may be easier for some readers to interpret. The contour plot shows the difference in the expected net present value of the two alternatives for pairs of values along the x and y-axes. Each contour is labeled to show the difference in the expected value of purchasing the charter fishing boat or holding the existing investment. The contours are isoclines, along which the difference in expected values of the outcomes are constant. For example, along the contour labeled \$10,000, the expected net present value of the charter fishing boat is \$10,000 higher than the alternative investment. The expected outcomes of the two alternatives are identical along the break-even contour, labeled \$0. Along the break-even contour, the decision maker is indifferent between the two alternatives. In Figure 15 (b), the x-axis is the probability that operating cost is high, p(Z=\$225/day). The y-axis is the probability that the booking rate is low, p(Y=0.4). For probabilities above the break-even contour, the alternative investment A_2 is preferred. For probabilities below the break-even contour, the charter fishing boat A_1 is preferred. The advantage of using contour plots to evaluate the sensitivity of

the decision is that the decision maker can readily see how the outcome of the decision might change over the range of probabilities he might assess at each chance node.

Sensitivity analysis of the expected monetary value is useful to a risk-neutral decision maker. For risk-averse decision makers like Rick Barton, outcome performance measures should be expressed in terms of utility scores and the decision should be made by maximizing utility. Likewise, the sensitivity analysis should examine the sensitivity of the utility scores. Figure 16 shows a contour plot sensitivity analysis for Rick Barton's charter fishing boat decision. This analysis is based on a risk tolerance parameter of 10,000. The contours are labeled with the difference in the utility score between the two alternatives. The dot, located at p(*Z*=\$225/day) = 0.5 and p(*Y*=0.4) = 0.52, is at the intersection of Rick Barton's probability assessments for the operating cost state Z=\$225/day and booking rate state Y = 0.4. The dot is above the isocline marked "0.0," indicating that, for that probability assessment, the optimal alternative is the alternate investment. The intersection is above the break-even isocline, so the optimal alternative is A2.

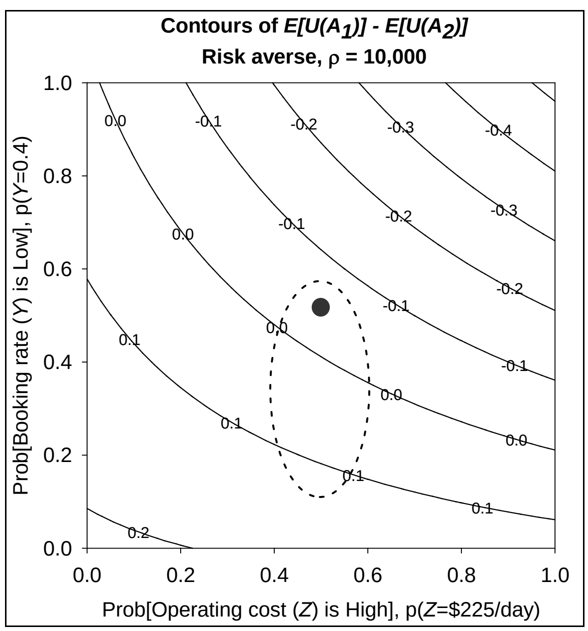


Figure 16. Sensitivity analysis for Rick Barton's charter fishing boat decision.

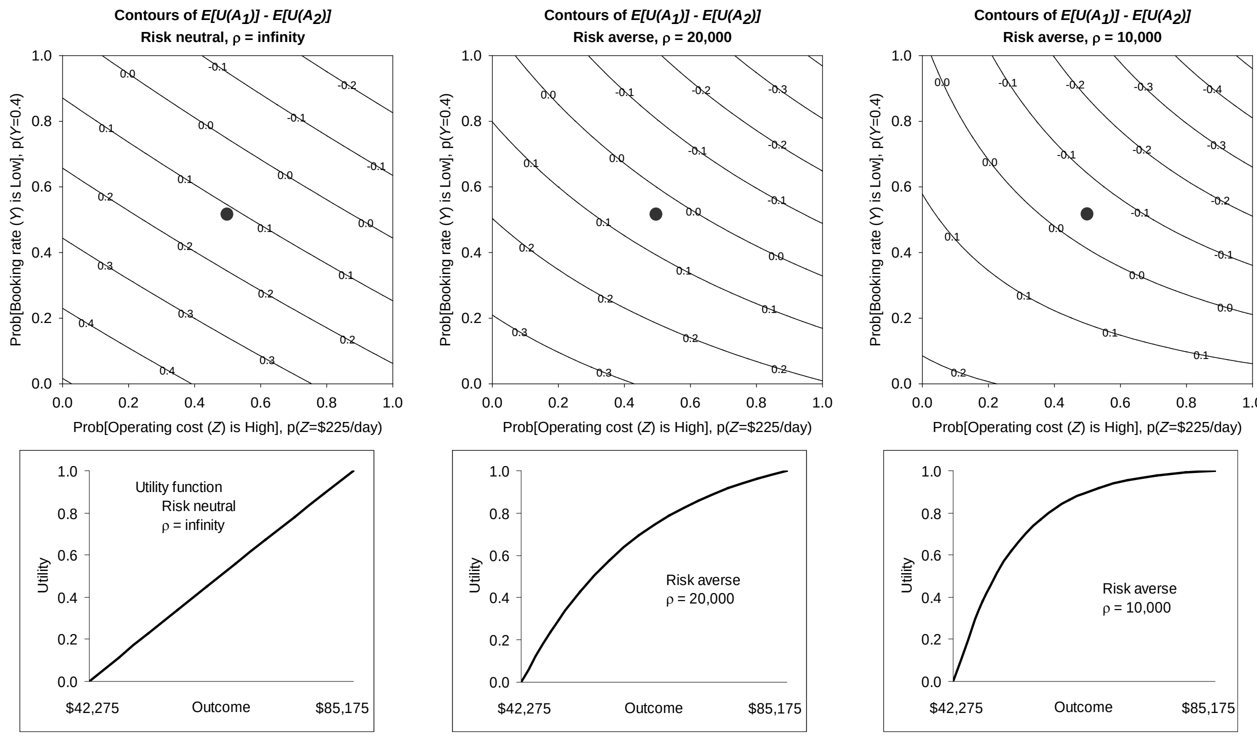
Rick's decision analyst friend points out that all of Rick's probability assessments were made based on data that were collected from other charter fishing boat operations while the economy was in a recession. He suggests that Rick might want to reassess the probabilities in his decision

tree considering that the economy could improve. The region outlined by a dashed line suggests where the intersection of new probabilities for operating cost and booking rate states might lie if economic conditions improved. This sensitivity analysis suggests how easily new information might change the decision. Rick also realizes that decision analysis doesn't tell the decision maker what to do, its real value to the decision maker is as an aide in learning about and understanding the decision problem.

Sensitivity to risk preferences

The key parameter in calculating a utility score is the risk tolerance parameter. While it is assumed that a decision maker knows his risk tolerance implicitly, it is often difficult to assess that value. In addition, when solving decision problems for third parties, it is often useful to evaluate the decision over a range of risk tolerances to illustrate how the decision depends upon the decision maker's own preferences. This enables a decision maker to align himself with the risk tolerance that fits him the best and is also useful in cases where there may be multiple stakeholders. Figure 17 illustrates three sensitivity analyses for the charter boat decision considering three levels of risk tolerance. Figure 17 (a) shows results of the sensitivity analysis for a risk neutral decision maker (ρ = infinity). Figures 17 (b) and 17 (c) replicate the sensitivity analysis for risk-averse decision makers. Figure 17 (b) assumes a risk tolerance of $\rho = 20{,}000$. Figure 17 (c) assumes a risk tolerance of $\rho = 10{,}000$, which is Rick Barton's estimated risk tolerance.

Figure 17 enables the decision maker to easily see how risk aversion might affect the decision. For $\rho = 10{,}000$, the alternate investment is preferred. The break-even isocline shifts upwards as risk tolerance increases. For example, if Rick's risk tolerance was 20,000, then the optimal alternative would have been to buy the charter fishing boat. A sensitivity analysis for a risk tolerance of infinity is also shown to represent a risk neutral decision maker. Compare this with Figure 15(b) to confirm that the contours for differences in utility are the same for differences in expected monetary value if the decision maker is risk neutral. These figures enable the decision maker to find the optimal decision considering his individual beliefs about y and z and his level of risk aversion. Given his discomfort with the outcome of the decision, Rick may wish to reconsider his risk tolerance.



(a) (b) (c)

Figure 17. Sensitivity analysis of Rick Barton's charter boat decision considering three levels of risk aversion. The figure shows that decision makers who have different risk tolerances may choose different alternatives, even when the facts surrounding the decision are the same.

Value of information

Based on his decision analysis, Rick Barton has concluded that he should not buy the charter fishing boat. This decision was based on expectations about operating costs, the quality of fishing, and booking rates and capacities, which were a function of the economic climate in the coming year. If the fishing is good, operating costs will be low and profits are likely to be high. If the economy is bad, the charter boat is likely to be booked at a lower rate and profits are likely to be low. Rick's decision analyst friend points out that there may be value in reducing the amount of uncertainty faced in the charter fishing boat decision. He suggests value of information analysis as a way to gauge how much potential value new information might have.

Rick decides to focus his value of information analysis on the booking rate and the percent capacity variables in his decision model because these variables are based on information that is readily obtainable. While Rick knows that no forecasts will ever be perfect, he calculates EVPYI to obtain an upper bound on what he should be willing to pay to obtain perfect

information on these uncertain variables. Applying the equation for EVPYI in Chapter 4, he calculates EVPYI for each of the uncertain variables. The operating cost has an EVPYI of $471. The booking rate has an EVPYI of $437. The percent capacity has an EVPYI of $270. Calculation of EVPYI enables one to rank the sources of uncertainties to determine which ones should be resolved first. Of the three uncertainties, operating cost has the highest value and booking rate has the second highest value. The amounts indicate the maximum that an investor should be willing to pay to fully resolve each of the sources of uncertainty in the decision model. However, perfect information is rarely available and imperfect information will be worth less than perfect information. If imperfect information can be obtained, it is also possible to assess the value of imperfect information; however, this requires information about the uncertainty in that information.

Adaptive management

Rick Barton finds an economist who has developed a method to predict charter fishing boat booking rates using economic indicators. A review of this economist's record shows that his economic forecasts accurately predict booking rates about 70 percent of the time. When he predicts booking rates will be low, booking rates turn out to be low 80 percent of the time. When he predicts that booking rates will be high, booking rates turn out to be high 60 percent of the time. This information is outlined in Figure 18. The economist informs Rick that his forecast for the upcoming year is that booking rates will be high. It is still not too late to purchase the charter boat. Rick Barton summarizes the information that he has and quickly updates the probabilities in his decision tree to reflect this new information, which will enable him to reevaluate his decision.

ECONOMIST'S PREDICTION	TRUE BOOKING RATE STATE	
	Low	High
Low	0.8	0.4
High	0.2	0.6

Figure 18. Information on the economist's performance with respect to forecasting charter boat booking rates.

The performance information provided by the economist cannot by itself be used to update the probabilities because it does not contain information on the base rates. The new information provided by the economist is the forecast of high booking rates. This information can be used to update the booking rate probabilities in Rick Barton's decision tree. The initial probabilities for booking rate states in the decision tree are the prior probabilities $p(y_k)$. These are the base rates of occurrence for low and high booking rate years. The new information (or evidence) is the economist's prediction for the upcoming year.

The probability of observing the booking rate state given the evidence, $p(y_k|e)$, is calculated using Bayes rule. Figure 18 gives the likelihood, which is the conditional probability of observing the evidence (e) given that y is in the k^{th} state, $p(e|y_k)$. Given the economist forecasts a high booking rate, the posterior probabilities are calculated as follows:

$$p(y_k = \text{Low} \mid \text{Forecast}=\text{"High"}) = \frac{(0.2 \times 0.52)}{(0.2 \times 0.52 + 0.6 \times 0.48)} = 0.265, \text{ and}$$

$$p(y_k = \text{High} \mid \text{Forecast}=\text{"High"}) = \frac{(0.6 \times 0.48)}{(0.2 \times 0.52 + 0.6 \times 0.48)} = 0.735$$

Were the economist to have forecasted a low booking rate, the posterior probabilities would have been calculated as follows:

$$p(y_k = \text{Low} \mid \text{Forecast}=\text{"Low"}) = \frac{(0.8 \times 0.52)}{(0.8 \times 0.52 + 0.4 \times 0.48)} = 0.684, \text{ and}$$

$$p(y_k = \text{High} \mid \text{Forecast}=\text{"Low"}) = \frac{(0.4 \times 0.48)}{(0.8 \times 0.52 + 0.4 \times 0.48)} = 0.316$$

Rick Barton updates his decision by substituting the posterior probabilities for his prior probabilities at the booking rate chance node. These probabilities are conditioned on the information obtained from the economist in the form of a booking rate forecast. Re-solving the decision tree, he finds that his utility is now maximized by the charter boat investment.

In this example, Rick Barton has applied the passive approach to adaptive management because the new information he uses to update his decision is not obtained through a structured experiment. Although the information is not obtained as part of the decision-making process, the forecasts he uses to update the decision tree are qualified by information on the past performance of the forecaster.

7 A Decision Analysis Example: The Town Harbor Dredging Decision

This example demonstrates the application of decision making under uncertainty to a dredging decision that is representative of one that might be encountered at a USACE district. The example demonstrates how this decision might be framed, how uncertainties might be characterized, and how the decision tree would be structured. The example illustrates how sensitivity analysis can be used to evaluate the robustness of the decision and learn how changes in modeling assumptions might alter the decision. Sensitivity analysis can be used to build confidence in the results of an analysis. For example, this can be done by revealing potential opportunity costs.

The decision problem

The USACE District Office maintains navigation channels to support waterborne transportation into and out of Town Harbor. The navigation route leading to Town Harbor consists of two reaches, as shown in Figure 19. Characteristics of the Lower and Upper Reaches are described in Table 1. The Lower Reach is 3 miles long, 600 ft wide, and has an authorized depth of 40 ft. The Upper Reach is 2 miles long, 500 ft wide, and has an authorized depth of 37 ft. Approximately 40 percent of the cargo transported in and out of Town Harbor is off-loaded or on-loaded in the Lower Reach. The remainder of the cargo must pass through the Upper Reach.

In previous years, the channels have been maintained at depths that are less than the authorized depths because of budget limitations. The Lower Reach has been maintained at a depth of 36 ft and the Upper Reach has been maintained at a depth of 33 ft. Surveys show the Lower Reach presently has a depth of 30 ft and the Upper Reach presently has a depth of 29 ft. For the upcoming dredging cycle, the operations manager has been asked to provide an economic justification for his choice of dredging alternative.

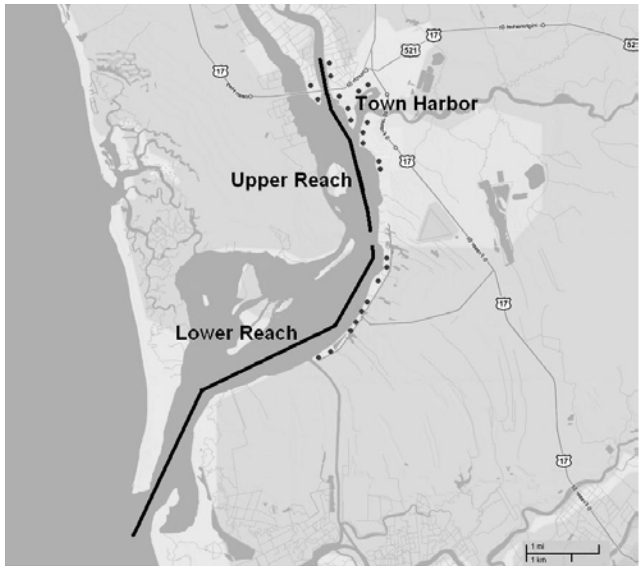


Figure 19. Map of Town Harbor showing Lower Reach and Upper Reach navigation channels.

Table 1. Navigation channel characteristics.

Characteristic	Lower Reach	Upper Reach
Channel length (miles)	3.0	2.0
Channel width (feet)	600	500
Authorized depth (feet)	40	37
Percent of tonnage	0.4	0.6
Survey depth (feet)	30	29
Unit cost of dredging (yard^{-3})	\$9.18	\$18.36

A dredging alternative is economically justified if the incremental cost of shipping is greater than the cost of dredging. The cost of shipping is defined as the cost per ton to transport cargo in and out of Town Harbor plus the maintenance dredging cost per ton of cargo transported. Shipping costs increase as the channel depth decreases because vessels that require deeper drafts must "adapt," meaning that the vessel must enter the harbor light-loaded, without using its full design draft, or anchor outside the port and lighter the cargo into port on barges. Both of these approaches raise the cost of shipping per ton of cargo.

The decision frame

The operations manager frames the decision by determining decision objectives and choosing project alternatives. The decision objective is to maximize the expected utility of the maintenance dredging program:

$$\underset{j}{\mathrm{MAX}}\ \mathrm{E}\left[\mathrm{U}\left(NPV_j\right)\right] \quad (9)$$

The function U is the utility function and NPV_j is the net present value of alternative j. MAX is an operator that means find alternative j that maximizes the utility function. For a risk-neutral decision maker, this objective is synonymous with minimizing the net present value of shipping and maintenance dredging costs per ton of cargo shipped in and out of Town Harbor during the dredging cycle. The net present value (NPV) of the j^{th} dredging alternative is the sum of shipping cost per ton during the maintenance period plus the maintenance cost per ton associated with the dredging alternative:

$$NPV_j = \sum_{t=1}^{T} \frac{1}{(1+r)^t} \left(\sum_{k=1}^{K} \alpha_k \overline{F}_k + \frac{C_j}{M} \right)_t \quad (10)$$

The average shipping cost per ton in Town Harbor is the sum of shipping cost per ton to the k^{th} reach $\overline{F}_k$ weighted by the proportion of tonnage destined for that reach α_k. The maintenance cost per ton is the total cost of the dredging alternative divided by the total tonnage transported in and out of Town Harbor. If dredging cycles span multiple years T, cost savings must be estimated for each year, discounted, and aggregated for the dredging cycle. For single period decisions in which all dredging costs are incurred in the same year that shipping cost savings are realized, discounting and aggregation of cost savings over multiple time periods can be neglected.

The operations manager uses shipping cost functions to estimate the average shipping cost per ton of cargo shipped in and out of Town Harbor. The shipping cost per ton in each reach is expressed as a function of the limiting depth in that reach:

$$F_{kj} = g\left(L_{kj}\right) = g\left(D_{kj} - S_k\right) \quad (11)$$

The cost of shipping is a function of the limiting depth L_{kj}, which is the difference between the dredge-to-depth D_{kj} and shoaling S_k during the maintenance period. Shoaling is the process by which sediment accumulates in the navigation channel, reducing the vessel drafts that can be accommodated in the channel. As the depth in the navigation channel decreases, the cost of shipping per ton of cargo delivered to Town Harbor

increases because the channel cannot accommodate vessels with deeper drafts, limiting the amount of cargo that can be loaded on each vessel. The shipping cost per ton is shown as a function of limiting depth in Figure 20. In this example, shipping cost per ton is expressed as a function of limiting depth because the decision variable is the dredge-to-depth. However, vessel size and the type of cargo being transported can be important factors in determining vessel operating costs and average shipping cost per ton. In this example, it is assumed that the fleet transporting cargo into and out of Town Harbor is uniform, as is the cargo.

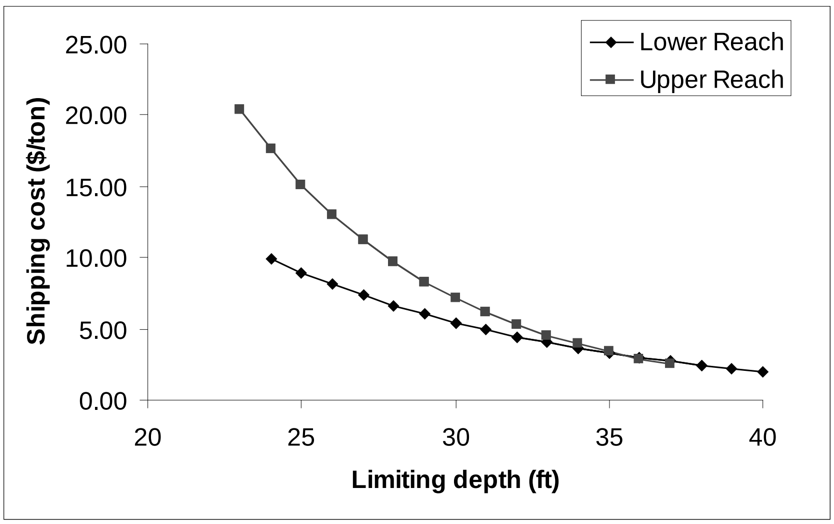


Figure 20. Shipping costs per ton as a function of limiting depth in the Upper Reach and the Lower Reach.

The maintenance dredging cost per ton of cargo shipped in and out of Town Harbor is the total cost of the dredging alternative divided by the market demand M. The cost of a dredging alternative is a function of the dredge-to-depth in each reach and the volume of material that must be dredged. The unit cost of dredging is determined by the type of material being dredged, the distance that must be traveled to disposal sites, and environmental protection constraints on dredging operations. Reliable estimates of unit dredging costs are available because all of these factors are known from surveys and dredging plans. Market demand, which is independent of dredging activities at Town Harbor, is used to calculate the average cost of maintenance dredging activities per ton of cargo shipped to and from Town Harbor. Market demand is uncertain and depends upon global economic factors.

Dredging alternatives are selected by the operations manager to represent a reasonably diverse set of possibilities from feasible regions of the decision space. Four feasible alternatives are identified for Town Harbor. These alternatives are outlined in Table 2. The operations manager is required to consider a no-action alternative (Alternative A_0) that would involve foregoing any dredging in both the lower and the upper reaches. The lower reach-only alternative (Alternative A_1) would emphasize dredging in the lower reach because the unit cost of dredging is lower than in the upper reach. The enhanced compromise plan (Alternative A_2) would dredge the lower reach to 38 ft and the upper reach to its authorized depth of 37 ft. The operations manager also considers a status quo plan (Alternative A_3) that would continue the past practice of dredging to 36 ft in the Lower Reach and to 33 ft in the upper reach.

Table 2. Town Harbor dredging alternatives.

Alternative	Description	Dredge-to-Depth (feet)		Cost of Alternative
		Lower Reach	Upper Reach	
A_0	No Action: Do not dredge navigation channels or perform maintenance at placement sites.	30	29	$0
A_1	Lower Reach Only: Dredge the lower reach to its authorized depth and forego dredging and maintenance in the upper reach.	40	29	$32,313,600
A_2	Enhanced Compromise: Dredge the lower reach to 38 ft and the Upper Reach to its authorized depth, 37 ft.	38	37	$54,574,080
A_3	Status Quo: Dredge the lower reach to 36 ft and dredge the upper reach to 33 ft.	36	33	$33,749,760

Sources of uncertainty

There are many sources of uncertainty in estimating the net benefits of each alternative. Shoaling levels in each reach vary from year to year as a function of local hydraulic conditions including precipitation, tides, currents, and the frequency and severity of storms. Shoaling during the upcoming dredging cycle is estimated using numerical models that account for these variables. The tonnage of imports to and exports from Town Harbor are predicted from historical records, but estimates are subject to uncertainty in global economic conditions and markets. The operations manager asks his staff to analyze uncertainty in those determinants of shipping cost savings that he regards as most likely to influence the decision: the shoaling rate in the upper reach (S_U), the shoaling rate in the lower reach (S_L), and market demand (M). Results of

the analysis are shown in Figure 21. The analysis shows that shoaling rates follow a lognormal distribution while market demand can be described by a normal distribution. The parameters for these distributions are shown in Table 3. The variables, which are continuous, are discretized for decision analysis. The operations manager selects the nominal values shown in Table 3 as midpoints of a discrete set of intervals that represent a set of possible events. The probability distributions are discretized to obtain the event probabilities shown in Table 4.

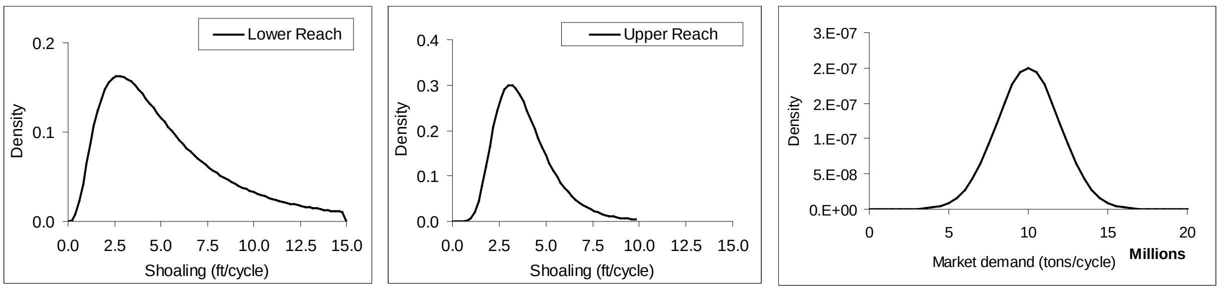


Figure 21. Uncertainty in key inputs to the net benefit function used to evaluate the performance of decision alternatives.

Table 3. Parameters and nominal values for uncertain variables considered in the decision.

Uncertain variable	Symbol	Parameters		Nominal values		
		Mean (μ)	Standard deviation (σ)	Low	Medium	High
Shoaling, Upper reach (ft/cycle)	S_U	4	4.8	2.5	5.0	7.5
Shoaling, Lower reach (ft/cycle)	S_L	6	1.6	2.0	4.0	6.0
Market demand (million tons/cycle)	M	10	2	5	10	15

Table 4. Event probabilities obtained by discretizing the probability distributions in Figure 14.

Shoaling in the Lower Reach (S_L)		Shoaling in the Upper Reach (S_U)		Market Demand (M)	
Midpoint (ft)	Probability	Midpoint (ft)	Probability	Midpoint ($)	Probability
2.5	0.3952	2	0.2901	5,000,000	0.1056
5	0.2978	4	0.4907	10,000,000	0.7887
7.5	0.3070	6	0.2192	15,000,000	0.1056

The decision model

The decision is modeled using a decision tree as shown in Figure 22. The figure shows four branches leading from the decision node, one for each alternative (A_0, A_1, A_2, and A_3). The decision tree expands for each alternative as shown by the branches emanating from A_j in the right-hand side of the figure. There are three chance nodes in the decision tree, one for each uncertain variable in the analysis. The probabilities at each chance node are based on a discretization of the continuous probability distributions for each uncertain variable. In this representation of the decision problem, each alternative leads to 27 possible outcomes. The decision outcomes are the net present value of shipping and maintenance dredging costs per ton of cargo shipped in and out of Town Harbor during the dredging cycle evaluated at the nominal values of the uncertain variables leading to that outcome. The expected NPV of each alternative, $E[NPV_j \mid A_j, p_i, y_i]$, is the probability weighted sum of the 27 potential outcomes for that alternative. The decision rule is to choose the alternative that minimizes the expected outcome.

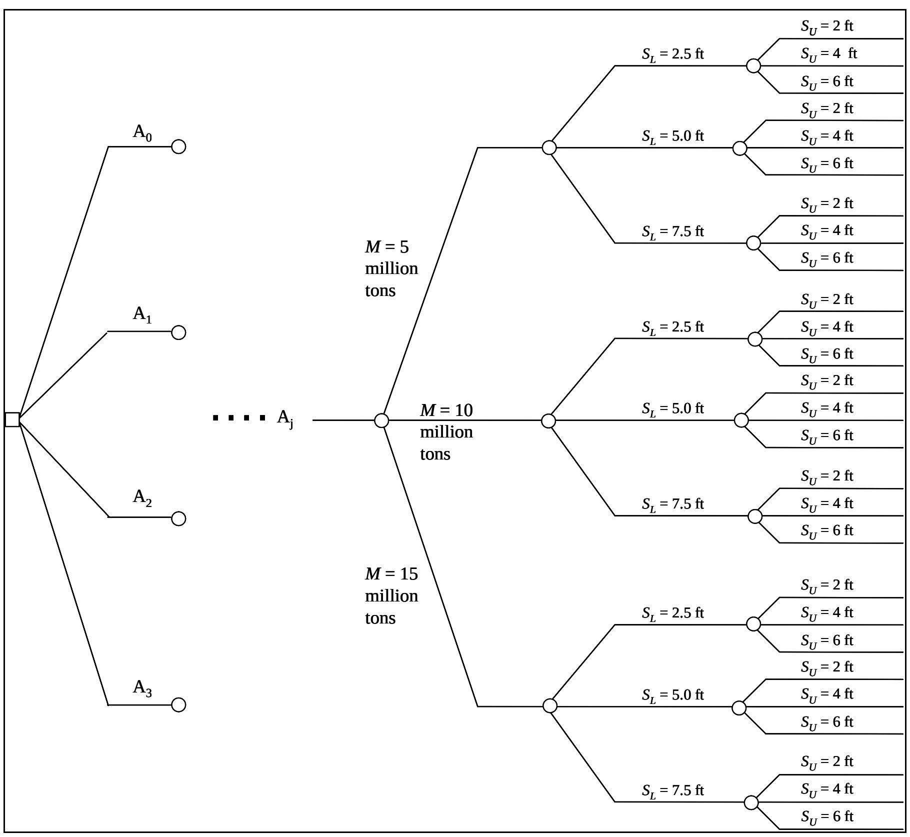


Figure 22. A decision tree for the Town Harbor dredging decision. The figure shows four dredging alternatives (A_0, A_1, A_2, and A_3) emanating from the decision node. There are 27 possible outcomes for each alternative.

Risk profiles

A risk profile is a cumulative probability distribution over all of the possible outcomes of an alternative. Its purpose is to characterize the uncertainty in the outcome of an alternative. Risk profiles for the Town Harbor dredging decision are illustrated in Figure 23. For each alternative, there are 27 possible realizations of the outcome, as illustrated by the decision tree in Figure 22. The risk profile in Figure 23 shows that, with all of the fixed parameters of the analysis at their nominal levels, the outcomes of the four dredging alternatives range from about $7.50 to about $20 per ton. The risk profile gives the probability that the shipping cost per ton is less than the corresponding amount on the *x*-axis. For example, under the enhanced compromise alternative (A_2), there is a 50% chance that the shipping cost per ton is less than $9.55 and a 90-percent confidence interval on shipping cost per ton would be $7.63 to $15.01. The status quo alternative (A_3) yields an outcome that is similar to A_2, but shipping costs per ton are likely to be slightly higher. In contrast, the lower reach-only alternative (A_1) performs much less well than the enhanced compromise alternative (A_2). The lower reach-only alternative leads to a 50-percent chance that the shipping cost per ton would exceed $13.33 and a 90-percent confidence interval on shipping costs that ranges from $10.57 to 16.86. Even the no-action alternative (A_0) performs better than the lower reach-only alternative.

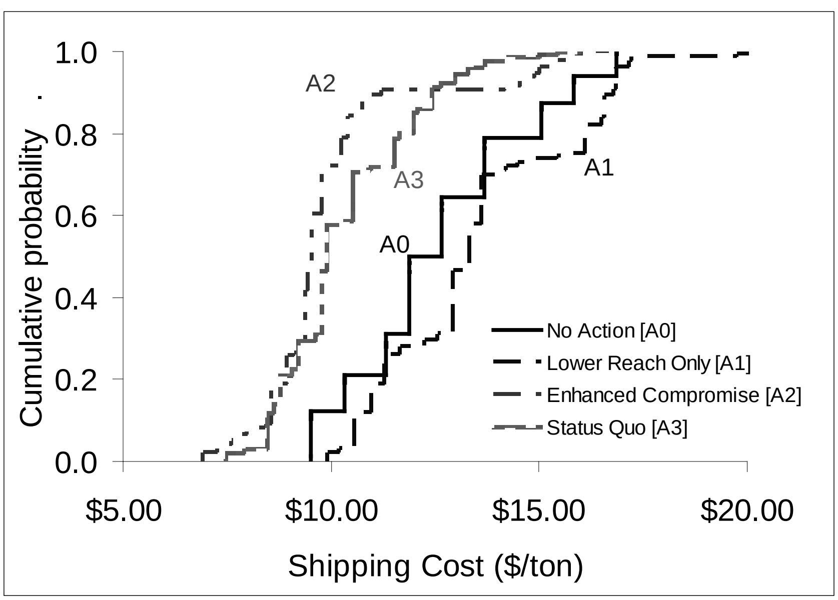


Figure 23. Risk profiles illustrate uncertainty in the net present value of potential outcomes for each alternative.

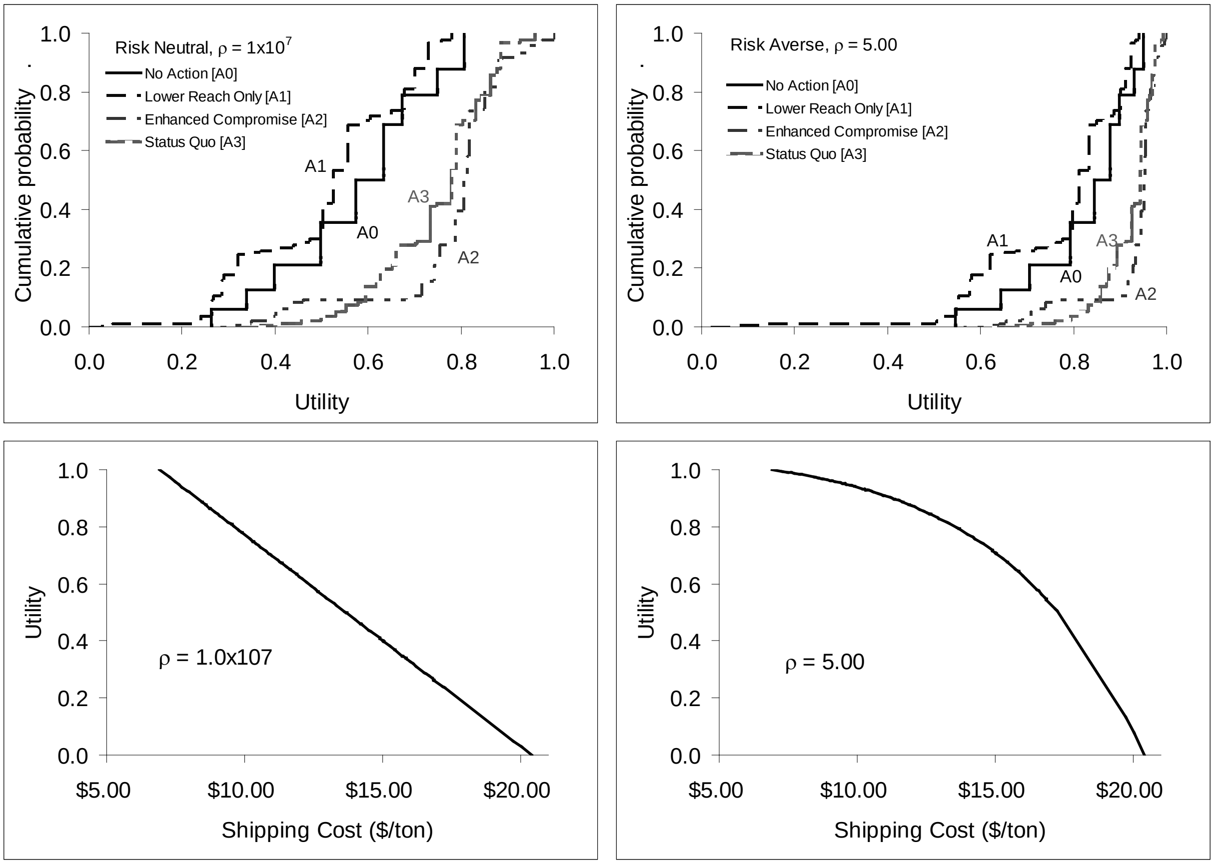
If a decision maker is risk neutral, the outcomes of a decision can be expressed in monetary or physical terms. However, if decision makers are risk averse, results should be expressed in terms of utility. A risk-averse decision maker places more emphasis on avoiding potential losses than achieving potential gains. The decision objective is to maximize expected utility (synonymous with minimizing the cost of shipping). In the Town Harbor dredging example, risk aversion translates into placing a higher value on reducing high shipping costs than on reducing low shipping costs. Risk aversion is modeled using a risk tolerance parameter ρ. Recall that the risk tolerance parameter is the maximum amount of a potential payout that would induce a decision maker to enter a lottery with a 50% chance of paying the entrant x and a 50% chance of costing the entrant $x/2$.

Maximizing utility rather than expected value will often lead a decision maker to a different set of conclusions. Figure 24 shows the risk profiles for the Town Harbor dredging decision in terms of utility. Results are displayed for a risk-neutral decision maker in (a), approximated by a risk tolerance parameter of $\rho = 1.0\text{x}10^7$. Results are displayed for a risk-averse decision maker in (b), with a risk tolerance of $\rho = 5.00$. In this case, risk aversion appears to have little effect on the decision. A_2 and A_3 lead to similar outcomes and A_1 is still dominated by A_0.

The expected utility of each alternative $E[U(NPV_j \mid A_j, y_i)]$ is the probability-weighted sum of the utility of monetary outcomes for that alternative. If the decision maker is risk neutral, maximizing utility is synonymous with minimizing expected shipping cost. Table 5 shows the expected net present value and utility scores for each alternative in the Town Harbor maintenance dredging example. A risk-neutral decision maker would choose A_2, the enhanced compromise alternative, because this alternative maximizes expected utility. A risk-averse decision maker characterized by a risk tolerance of 5.00 would also choose A_1.

The sensitivity analysis

The results of a decision analysis depend upon the parameter values and probability distributions that are used in modeling the decision. As with all models, it is useful to assess the sensitivity of modeling results both as a means to validate the results by ensuring that the results make sense and to inform the decision maker about key sensitivities in the decision. Sensitivity analyses should build confidence in an analysis supporting a decision or should reveal what aspects of an analysis might need



(a) (b)

Figure 24. Risk profiles showing uncertainty in the utility of outcomes for a risk-neutral decision maker with risk tolerance of infinity (approximated by $1.0x10^7$) (a) and a risk-averse decision maker with a low risk tolerance of 5.00 (b). Utility functions for each decision maker are displayed in the panels below the risk profiles.

Table 5. Expected *NPV* and expected utility for maintenance dredging alternatives.

Decision criteria	Risk tolerance parameter	Risk attitude	Units	Alternative			
				A_0	A_1	A_2	A_3
E[*NPV* | A_j , y_i]	-	-	($/ton)	12.67	13.87	10.19	10.57
E[U(*NPV* | A_j, y_i)]	$1.0x10^7$	Risk neutral	Utils	0.573	0.485	0.757	0.729
E[U(*NPV* | A_j, y_i)]	5.00	Risk averse	Utils	0.822	0.749	0.919	0.914

additional work. Sensitivity analyses are conducted by varying the parameters of the decision model and evaluating the net present value and utility of decision alternatives while holding all other parameters in the decision model constant. Results of the sensitivity analysis are displayed in the form of figures that show which alternatives are optimal for different combinations of parameter values. While the figures in this example are best viewed in color, they are designed so that they can also be read in black and white.

Several examples of sensitivity analyses are considered to demonstrate sensitivity analysis in different types of decision environments. This example considers sensitivity analysis for the single decision maker and sensitivity analysis for the collaborative decision maker. There are subtle differences in the approach. As a practical matter, a single decision maker should conduct sensitivity analysis only on the fixed variables and parameters used in the decision model. There is no reason to conduct sensitivity analysis on value parameters or probabilities because the decision maker knows his values and preferences and subjective probabilities are not uncertain. In contrast, a collaborative decision maker may be interested in conducting sensitivity analysis of value parameters and probabilities as a way of considering how the results might change, given different points of view of stakeholders involved in the decision process.

Sensitivity analysis for a single decision maker

For decisions involving a single stakeholder, the sensitivity analyses might be driven by concern over what nominal values have been used for fixed variables and parameters in the decision model. The question to be addressed is whether or not the optimal alternative changes over the range of plausible values. These analyses also help decision makers to develop an understanding of how their strategies might change in the event of changes in uncontrollable variables. This example considers sensitivity to two fixed variables, the unit cost of dredging in the upper reach and the unit cost of dredging in the lower reach. The unit cost of dredging is determined by the type of material being dredged, the distance that must be traveled to disposal sites, environmental protection constraints on dredging operations, and fuel prices. While there may be some correlation between these two variables, it is also possible that one has been over-estimated and the other has been under-estimated. Therefore, the variables are treated independently.

Sensitivity of the dredging decision to the unit cost of dredging in each reach is presented in Figure 25. The horizontal axis of the table shows a range of possible unit dredging costs in the lower reach. The vertical axis shows a range of possible unit dredging costs in the upper reach. Each cell of the table indicates the decision alternative that would be optimal under some combination of unit dredging costs in each reach. This figure helps to validate the decision model because it shows that alternatives that require more dredging activity are only optimal when unit dredging costs are low. As unit dredging costs increase, the no-action alternative (A0) becomes optimal. Showing results like this should help to build confidence in the decision model.

Results in Figure 25(a) are for a risk-neutral decision maker. The nominal unit dredging costs in Town Harbor are \$9.18 yd^{-3} in the lower reach and \$18.36 yd^{-3} in the upper reach. While these costs are treated as fixed variables in the decision model, they may not be devoid of uncertainty. In addition, these costs can change over time depending upon factors beyond the control of the decision maker; therefore, it is useful to have insight into how such changes might alter the decision. These results show that the decision has a high degree of sensitivity to the nominal unit costs assumed in this analysis. The preferred alternative for the nominal costs is the enhanced compromise alternative (A2). However, a relatively minor increase in the unit cost of dredging might make the status quo alternative (A3) more preferable.

In Figure 25(b), the sensitivity to unit dredging costs is shown for a risk-averse decision maker to show how the set of optimal decisions might be affected by risk attitudes. In this case, a risk tolerance parameter of $\rho = 5.0$ is used to simulate risk aversion. There are some differences between figures 25(a) and 25(b). For example, the optimal alternative at the intersection of \$20.00/yd^3 for the lower reach and \$10.00/yd^3 for the upper reach switches from A2 for the risk-neutral decision maker to A3 for the risk averse decision maker. Despite a few differences like these, the comparison of Figures 25(a) and 25(b) reveals that, at least with respect to these variables, this decision has rather limited sensitivity to the decision maker's risk attitudes.

Dredging Cost - Lower Reach ($/yd^3)

Dredging Cost - Upper Reach ($/yd^3)	0.00	2.50	5.00	7.50	10.00	12.50	15.00	17.50	20.00	22.50	25.00
0.00	A2	A2	A2	A2	A2	A2	A2	A2	A2	A2	A2
5.00	A2	A2	A2	A2	A2	A2	A2	A2	A2	A2	A0
10.00	A2	A2	A2	A2	A2	A2	A2	A2	A2	A0	A0
15.00	A2	A2	A2	A2	A2	A2	A2	A2	A0	A0	A0
20.00	A2	A2	A2	A2	A2	A3	A3	A3	A0	A0	A0
25.00	A2	A2	A2	A3	A3	A3	A3	A0	A0	A0	A0
30.00	A2	A3	A3	A3	A3	A3	A0	A0	A0	A0	A0
35.00	A3	A3	A3	A3	A3	A0	A0	A0	A0	A0	A0
40.00	A3	A3	A3	A3	A3	A0	A0	A0	A0	A0	A0
45.00	A1	A3	A3	A3	A0	A0	A0	A0	A0	A0	A0
50.00	A1	A1	A3	A0	A0	A0	A0	A0	A0	A0	A0

(a) Sensitivity analysis for a risk-neutral decision maker, $\rho = 1.0x10^7$

Dredging Cost - Lower Reach ($/yd^3)

Dredging Cost - Upper Reach ($/yd^3)	0.00	2.50	5.00	7.50	10.00	12.50	15.00	17.50	20.00	22.50	25.00
0.00	A2	A2	A2	A2	A2	A2	A2	A2	A2	A2	A2
5.00	A2	A2	A2	A2	A2	A2	A2	A2	A2	A2	A0
10.00	A2	A2	A2	A2	A2	A2	A2	A2	A3	A0	A0
15.00	A2	A2	A2	A2	A2	A2	A3	A3	A0	A0	A0
20.00	A2	A2	A2	A2	A3	A3	A3	A3	A0	A0	A0
25.00	A2	A2	A3	A3	A3	A3	A3	A0	A0	A0	A0
30.00	A3	A3	A3	A3	A3	A3	A0	A0	A0	A0	A0
35.00	A3	A3	A3	A3	A3	A0	A0	A0	A0	A0	A0
40.00	A3	A3	A3	A3	A3	A0	A0	A0	A0	A0	A0
45.00	A1	A3	A3	A3	A0	A0	A0	A0	A0	A0	A0
50.00	A1	A1	A1	A0	A0	A0	A0	A0	A0	A0	A0

(b) Sensitivity analysis for a risk-averse decision maker, $\rho = 5.00$

Figure 25. Sensitivity of the decision to unit dredging costs in the Lower and Upper Reaches. Sensitivity is evaluated for a risk-neutral decision maker (a) and a risk-averse decision maker (b).

Sensitivity analysis for collaborative decision makers

In collaborative decision-making processes, sensitivity analyses can be used as a way of confronting stakeholder differences and achieving consensus around common courses of action. In this context, it makes sense to consider differences among stakeholders with respect to assumptions about fixed variables in the analysis, subjective probability assessments, and risk attitudes. The question to be addressed is whether or not different assumptions with regard to these aspects of the decision model would lead different decision makers to select different courses of action. If not, concerns or differences of opinion may be overcome, leading to consensus for a common alternative. If so, this provides a way of focusing deliberation on the relevant issues. These types of sensitivity analyses can also be useful when analysts are presenting results to third-party decision makers.

It is not at all unreasonable to expect that different stakeholders might have different levels of risk aversion. Therefore, the risk tolerance parameter in this example is a key variable of interest. Another key variable is the expected market demand, which is a fixed parameter of the

distribution characterizing uncertainty in the market demand variable. Figure 26 shows a sensitivity analysis for these two variables. As in Figure 25, each cell indicates which alternative is preferred given the combination of risk tolerance parameter and expected market demand assumed in the analysis. The figure shows that, at the nominal value of market demand (10 million tons), the decision is relatively insensitive to differences in risk attitudes, particularly at levels of risk tolerance greater than ρ = 4.0. For decision makers who are highly risk averse (ρ < 5.0), the decision shows some sensitivity to risk tolerance.

	Risk Tolerance Parameter, ρ										
Expected market tonnage, E[*M*] (million tons)	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	$1.0x10^7$
5.0	A3	A0	A0	A0	A0	A0	A0	A0	A0	A0	A0
5.5	A3	A0	A0	A0	A0	A0	A0	A0	A0	A0	A0
6.0	A3	A0	A0	A0	A0	A0	A0	A0	A0	A0	A0
6.5	A3	A3	A0	A0	A0	A0	A0	A0	A0	A0	A0
7.0	A3	A3	A3	A3	A3	A3	A3	A3	A3	A3	A3
7.5	A3	A3	A3	A3	A3	A3	A3	A3	A3	A3	A3
8.0	A3	A3	A3	A3	A3	A3	A3	A3	A3	A3	A3
8.5	A3	A3	A3	A3	A3	A3	A3	A3	A3	A3	A3
9.0	A3	A3	A3	A3	A3	A3	A3	A3	A3	A3	A2
9.5	A3	A3	A3	A3	A2	A2	A2	A2	A2	A2	A2
10.0	A3	A3	A3	A2	A2	A2	A2	A2	A2	A2	A2
10.5	A3	A3	A2	A2	A2	A2	A2	A2	A2	A2	A2
11.0	A3	A2	A2	A2	A2	A2	A2	A2	A2	A2	A2
11.5	A3	A2	A2	A2	A2	A2	A2	A2	A2	A2	A2
12.0	A3	A2	A2	A2	A2	A2	A2	A2	A2	A2	A2
12.5	A3	A2	A2	A2	A2	A2	A2	A2	A2	A2	A2
13.0	A3	A2	A2	A2	A2	A2	A2	A2	A2	A2	A2
13.5	A3	A2	A2	A2	A2	A2	A2	A2	A2	A2	A2
14.0	A3	A2	A2	A2	A2	A2	A2	A2	A2	A2	A2
14.5	A3	A2	A2	A2	A2	A2	A2	A2	A2	A2	A2
15.0	A3	A2	A2	A2	A2	A2	A2	A2	A2	A2	A2
15.5	A2	A2	A2	A2	A2	A2	A2	A2	A2	A2	A2

Figure 26. Sensitivity of the dredging decision to risk tolerance and expected market tonnage.

Figure 26 also shows that differences in beliefs about expected market demand can have an effect on which alternative is optimal. At higher levels of risk tolerance, the no-action alternative (A0) tends to be preferred when expected market demand is low (below 6.5 million tons). The status quo alternative (A3) tends to be preferred if expected market demand is between 7.0 and 9.5 million tons. The enhanced compromise alternative (A2) is preferred when expected market tonnage is greater than 9.5 million metric tons. These conclusions show sensitivity to risk attitudes. As risk tolerance decreases (ρ < 5.0), the status quo alternative (A3) is optimal over an increasingly large range of expected market demand. This analysis lets stakeholders who have differing expectations about market demand and different attitudes toward risk assess whether these differences would lead to different courses of action.

The fraction of tonnage headed to and from the upper reach is an important fixed variable in the decision model because the high cost of dredging in the upper reach can only be justified if a sufficient tonnage will pass

through the Upper Reach. The nominal fraction of tonnage headed to the Upper Reach is 0.6. Figure 27 shows the sensitivity of the dredging decision to this variable and the risk tolerance parameter. When the fraction of tonnage headed to the upper reach is less than or equal to about 0.2, the lower reach-only alternative (A1) is preferred. This result is consistent for all levels of risk tolerance. If the fraction of tonnage bound to or from the Upper Reach is greater than 0.2, either the enhanced compromise alternative (A3) is preferred or the lower reach-only alternative (A2) is preferred. The optimal alternative switches from A3 to A2 as the fraction increases. At higher levels of risk aversion ($\rho < 5.0$), the decision shows greater sensitivity to the risk tolerance parameter.

Fraction of Tonnage to/from Upper Reach	Risk Tolerance Parameter, ρ										
	1.0	**2.0**	**3.0**	**4.0**	**5.0**	**6.0**	**7.0**	**8.0**	**9.0**	**10.0**	**$1.0x10^7$**
0.00	A1	A1	A1	A1	A1	A1	A1	A1	A1	A1	A1
0.10	A1	A1	A1	A1	A1	A1	A1	A1	A1	A1	A1
0.20	A1	A1	A1	A1	A1	A1	A1	A1	A1	A1	A1
0.30	A3	A3	A3	A3	A3	A3	A3	A3	A3	A3	A3
0.40	A3	A3	A3	A3	A3	A3	A3	A3	A3	A3	A3
0.50	A3	A3	A3	A3	A3	A3	A3	A3	A3	A2	A2
0.60	A3	A3	A3	A2	A2	A2	A2	A2	A2	A2	A2
0.70	A3	A2	A2	A2	A2	A2	A2	A2	A2	A2	A2
0.80	A3	A2	A2	A2	A2	A2	A2	A2	A2	A2	A2
0.90	A2	A2	A2	A2	A2	A2	A2	A2	A2	A2	A2
1.00	A2	A2	A2	A2	A2	A2	A2	A2	A2	A2	A2

Figure 27. Sensitivity of the decision to risk tolerance and the fraction of tonnage headed to and from the upper reach.

As discussed above, the maintenance dredging decision is sensitive to both the expected market demand (M) and the fraction of cargo transported to and from the upper reach. The sensitivity to these variables may depend upon the decision maker's risk tolerance. Figure 28 explores how the decision might change for different values of these variables at two levels of risk aversion. Figure 28(a) shows results for a risk-neutral decision maker. For low levels of market demand ($E[M] \leq 7$), no action (A0) is preferred unless a relatively large fraction of the tonnage is bound to and from the upper reach, in which case the status quo alternative (A3) tends to be preferred. At higher levels of market demand ($M \geq 8$), A1, A2, and A3 are preferred, again depending upon what fraction of tonnage is bound for the Upper Reach. This pattern shows some sensitivity to risk aversion in 21(b), as indicated by the expanded region over which A3 is preferred. However, this sensitivity to risk attitude seems relatively minor.

Expected Market Tonnage, E[*M*] (Million tons)

Fraction of Tonnage to/from Upper Reach	5	6	7	8	9	10	11	12	13	14	15
0.00	A0	A0	A1	A1	A1	A1	A1	A1	A1	A1	A1
0.10	A0	A0	A0	A1	A1	A1	A1	A1	A1	A1	A1
0.20	A0	A0	A0	A1	A1	A1	A1	A1	A1	A1	A1
0.30	A0	A0	A0	A3	A3	A3	A3	A3	A2	A2	A2
0.40	A0	A0	A0	A3	A3	A3	A2	A2	A2	A2	A2
0.50	A0	A0	A0	A3	A3	A2	A2	A2	A2	A2	A2
0.60	A0	A0	A3	A3	A2	A2	A2	A2	A2	A2	A2
0.70	A0	A0	A3	A2	A2	A2	A2	A2	A2	A2	A2
0.80	A0	A0	A3	A2	A2	A2	A2	A2	A2	A2	A2
0.90	A0	A3	A2	A2	A2	A2	A2	A2	A2	A2	A2
1.00	A3	A3	A2	A2	A2	A2	A2	A2	A2	A2	A2

(a) Sensitivity analysis for a risk-neutral decision maker, $\rho = 1.0\text{x}10^7$

Expected Market Tonnage, E[*M*] (Million tons)

Fraction of Tonnage to/from Upper Reach	5	6	7	8	9	10	11	12	13	14	15
0.00	A0	A0	A1	A1	A1	A1	A1	A1	A1	A1	A1
0.10	A0	A0	A0	A1	A1	A1	A1	A1	A1	A1	A1
0.20	A0	A0	A0	A1	A1	A1	A1	A1	A1	A1	A1
0.30	A0	A0	A0	A3	A3	A3	A3	A3	A3	A2	A2
0.40	A0	A0	A0	A3	A3	A3	A3	A2	A2	A2	A2
0.50	A0	A0	A0	A3	A3	A3	A2	A2	A2	A2	A2
0.60	A0	A0	A3	A3	A3	A2	A2	A2	A2	A2	A2
0.70	A0	A0	A3	A3	A2	A2	A2	A2	A2	A2	A2
0.80	A3	A3	A3	A2	A2	A2	A2	A2	A2	A2	A2
0.90	A3	A3	A3	A2	A2	A2	A2	A2	A2	A2	A2
1.00	A3	A3	A2	A2	A2	A2	A2	A2	A2	A2	A2

(b) Sensitivity analysis for a risk-averse decision maker, $\rho = 5.00$

Figure 28. Sensitivity of the decision to expected market tonnage and the fraction of tonnage headed to and from the upper reach for a risk-neutral decision maker (a) and a risk-averse decision maker (b).

Sensitivity analysis and potential opportunity costs

The sensitivity analyses described above show how the decision analysis might lead to different conclusions given different assumptions in the decision model and given different stakeholder values and beliefs. Those sensitivity analyses did not reveal any information about what opportunity costs might be incurred as a result of choosing a sub-optimal alternative. Understanding what the opportunity costs might be will suggest how much effort a decision maker should expend to avoid choosing a sub-optimal alternative. For example, if the opportunity costs of choosing the enhanced compromise alternative (A2) instead of the status quo alternative (A3) are relatively low, it may not really matter whether the optimal alternative or the second-best alternative is chosen, since the potential opportunity costs are small.

Opportunity costs can only be interpreted correctly with respect to fixed variables in the analysis. Although fixed variables should have been chosen because of the uncertainty about what value should be used for that variable, those variables may still be associated with some uncertainty. Opportunity costs are defined as the difference between the expected outcome of the chosen alternative and the expected outcome of the

alternative that minimizes expected shipping cost per ton, evaluated at the realized value of the fixed variable in question. Because the realized value of a variable cannot be known a priori, it is useful to think of these analyses as revealing potential opportunity costs. In this example, potential opportunity costs are evaluated in Figure 29(a) with respect to the fraction of market demand bound to and from the upper reach and in Figure 29(b) for a unit dredging cost scaling parameter. These results are presented for a risk-neutral decision maker. The outcomes of the decision are expressed in unit shipping costs; therefore, the total potential opportunity costs in this example must be weighted by the expected market demand. For example, a difference of $1 in shipping cost per ton between a chosen alternative and an optimal alternative translates into a potential opportunity of $10 million.

Figure 29(a) shows that, if the fraction of tonnage bound to and from the upper reach is held at its nominal value, 0.6, the enhanced compromise alternative (A2) minimizes the expected NPV of shipping cost per ton. Above the nominal value of 0.6, there are no potential opportunity costs because A2 remains the preferred alternative. If the fraction of tonnage bound to and from the upper reach drops below about 0.45, the optimal alternative switches over to the status quo alternative (A3). If the fraction of tonnage bound to and from the upper reach drops below about 0.2, the optimal alternative switches over to the lower reach-only alternative (A1). The potential opportunity cost of choosing A2 is the difference between the shipping cost per ton under A2 and the shipping cost per ton under the optimal alternative evaluated at the realized fraction of tonnage bound to and from the upper reach. These results suggest that, unless the nominal fraction of tonnage bound to and from the upper reach is over-estimated by more than 0.15, there will be no potential opportunity costs as a result of choosing the enhanced compromise alternative (A2).

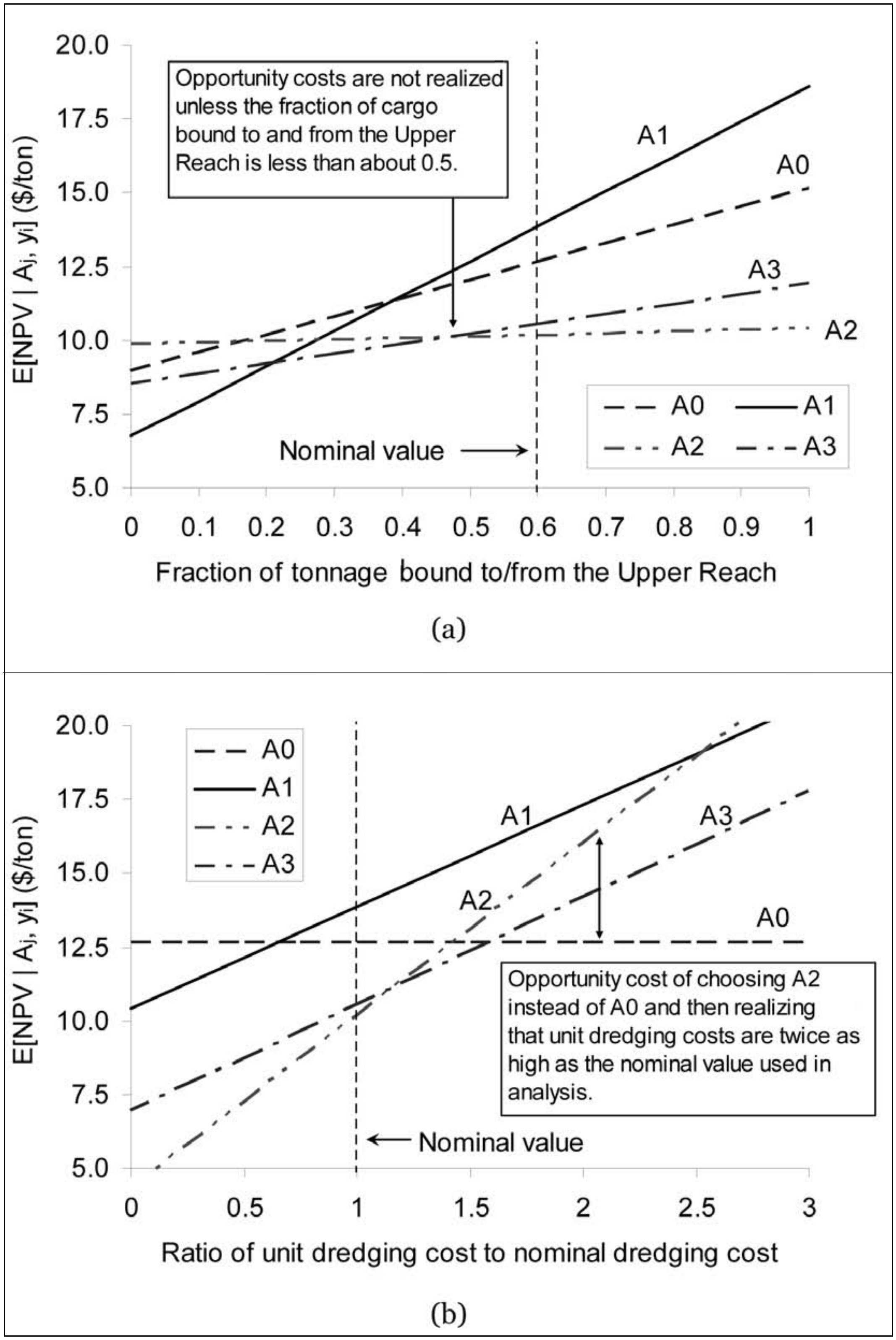


Figure 29. Sensitivity analysis and evaluation of potential opportunity costs of the decision to the unit cost of dredging showing the opportunity cost of choosing A2 over A3 and then realizing unit dredging costs that are twice as high as expected because of unexpected fuel price increases.

The sensitivity analysis above showed that the decision is sensitive to the unit cost of dredging in each reach. These two variables are partially independent. Therefore, this sensitivity analysis and evaluation potential opportunity costs is with respect to a scaling parameter unit dredging costs. Changes in the scaling parameter imply proportional changes in unit dredging cost. Results of this analysis can be seen in Figure 29, which shows that A2 minimizes the expected shipping cost per ton when the ratio of dredging cost to nominal dredging cost is 1.0. If the unit cost of dredging is below 1.0, A2 remains the preferred alternative. However, if the unit cost of dredging is higher than the nominal value, the status quo alternative (A3) minimizes the expected outcome for ratios between 1.2 and about 1.7. Above ratios of 1.7, the no-action alternative (A0) minimizes the expected outcome of the decision.

Over- or under-estimating the unit dredging costs in the decision model could lead the decision maker to choose a sub-optimal alternative. This would lead to opportunity costs. For example, the opportunity cost of choosing A2 instead of A0 and then realizing unit dredging costs will be twice as high as expected is $3.14 per ton, which translates into a total opportunity cost of $314 million, given the expected market demand of 10 million. There appear to be no potential opportunity costs as long as unit dredging costs stay below 125% of the nominal unit costs used in the decision model. As with other forms of sensitivity analyses, assessments of potential opportunity costs should build a decision maker's confidence in the results of a decision analysis by creating familiarity with the decision landscape. However, if the decision is overly sensitive to the variables of interest or the potential opportunity costs are too high, these analyses will reveal where additional work may be needed to improve the decision model or clarify which decision alternatives might be preferred by the decision maker.

Summary of the Town Harbor example

The Town Harbor dredging example has illustrated the process of decision making under uncertainty and demonstrated some of the principles and techniques that were discussed in the main body of the report. Specifically, this example demonstrates the following procedures for decision making under uncertainty:

1. Frame the decision problem. The decision problem is framed by identifying a decision objective and decision alternatives. In this example,

the objective is to maximize expected utility during the dredging cycle. Assuming risk neutrality, this is the same as minimizing shipping cost per ton. Four decision alternatives are considered.

2. Identify sources of uncertainty. Three key sources of uncertainty are identified as relevant to this decision. They include the shoaling rate in the lower reach, the shoaling rate in the upper reach, and market demand, which is defined as expected tonnage during the dredging cycle.
3. Create a decision model. A decision tree is developed to model the decision problem. Each of the uncertainties is discretized to three possible levels. Twenty-seven possible outcomes for each alternative are enumerated. The probability of each uncertain variable state is obtained by discretizing a continuous probability distribution that characterizes uncertainty in the continuous uncertain variable.
4. Evaluate the potential outcomes. The potential outcomes and their probabilities are evaluated for each alternative and risk profiles are constructed. The example compares risk profiles for a risk-neutral decision maker with risk profiles for a risk-averse decision maker.
5. Conduct a sensitivity analysis. Sensitivity analysis is used to evaluate the sensitivity of the decision. For individual decision makers, the sensitivity analysis revolves around fixed variables in the decision model. For collaborative decision makers, the sensitivity analysis may also revolve around value parameters and subjective probability assessments. Potential opportunity costs are evaluated to assess the potential costs of choosing a sub-optimal alternative.

8 Implementing Decision Making under Uncertainty in Practice

The concepts and approaches presented within this report have been applied to a range of decision problems and have been taught as a part of engineering, economics and management science curricular for decades (*e.g.*, Howard et al. 1972; Hobbs et al. 1997; Lund 2008). Why have these approaches not been more widely adopted in government agencies and other organizations confronting the challenges of decision making under uncertainty? Lund (2008) attributes this gap to institutional and individual reluctance to change; skepticism toward unfamiliar concepts; the role that politics plays in decision making; the perception that the methods are too difficult or resource intensive to implement; and fears about having too much, too little, or inappropriate data. Barriers to the adoption of decision analysis methods in USACE can be overcome by assisting analysts to become more familiar with the methods and concepts and encouraging their use in agency decisions.

There is no substitute for gaining hands-on experience solving decision problems. Useful guidelines for organizing and implementing probabilistic decision analyses may assist decision makers to gain experience with decision making under uncertainty methods. These include:

- *Incorporate decision analysis from the very earliest stages of the project.* The efficacy of probabilistic decision analysis often depends upon the initial framing of the decision problem, which outlines the decision objectives and alternatives. The framing of a decision problem will influence choices about what uncertainties to address and what kinds of data to collect. What data are collected and how those data are collected will constrain the type of analysis that can be performed. Therefore, the decision to use probabilistic decision analysis methods should be made as soon as possible in the decision-making process.
- *Scale the investments in analysis to match the decision.* As demonstrated in this report, probabilistic decision methods can be implemented effectively without major investments. Generally, the level of investment in a probabilistic decision analysis should reflect the importance of the decision (Lund 2008). Larger investments in decision analysis are justified by higher opportunity costs.

- *Use a tiered, iterative approach to decision analysis.* A tiered approach starts with a screening level analysis that provides a preliminary sketch of the decision problem using available data or rough approximations. The active thinking that is required to do a thoughtful screening-level analysis will assist the decision maker to flush out alternatives, adapt the decision frame, and identify decision-relevant variables and uncertainties. At the screening level, it is the process rather than the results that are most important. Results of the screening-level analysis can also provide an indication of opportunity costs and whether additional analysis may be justified. This sets the decision maker up for another iteration of the analysis incorporating what was learned in the previous stage, including any re-framing of the decision problem and any improvements in the quality of data used in the analysis.
- *Stop the analysis when there is sufficient confidence to make the decision.* It is possible for the analysis of a decision problem to continue beyond the point that it has value for the decision maker. A decision analysis should continue only until the decision maker has the confidence he needs to make a decision. There is no uniform test for determining when that point has been reached. Confidence in an analysis is gained through critical evaluation of the information and methods used in modeling the decision and through sensitivity analysis. Each decision maker will reach this point of confidence at a different point in the deliberation process, depending upon the extent of his insight into the decision problem and his personality.

The guidelines described here anticipate and encourage experimentation in solving difficult decision problems. An advantage of allowing decision makers to experiment is that it is likely to spur innovation and help decision makers to identify problem- and site-specific issues that are relevant to the decisions they make. However, experimentation may tend to generate a fair number of analyses that are poorly done. The advantage of canned analytical approaches is that they encourage consistency across the agency. As more experience is gained with the application of decision analysis and methods of applying these techniques to agency decisions are refined, a transition to prescribed decision methods should be possible.

Do risk and decision analysis support USACE priorities and requirements?

Probabilistic risk and decision analysis methods, as described in this report, satisfy USACE requirements for decision making. The USACE Campaign Plan repeatedly calls for the use of risk-based and risk-informed approaches to guide investment decisions and risk management actions. Two notable examples of this are:

- **Objective 3b: Improve resilience and lifecycle investment in critical infrastructure.** USACE will improve resiliency of critical infrastructure to reduce risks to critical water resources and infrastructure critical to DoD from an all hazards systems approach, to include hostile activity. Improved resilience of critical infrastructure ensures availability of networked assets critical to the nation. Investment decisions must be risk-based and meet the priorities of the component programs…
- **Objective 3c: Deliver reliable infrastructure using a risk-informed asset management strategy.** USACE will deliver a reliable infrastructure to ensure these assets continue to provide value to the Nation and meet expected levels of service while mitigating risk. Increased reliability will be achieved by developing a strategy, which includes an integrated national plan for assessing the infrastructure and an investment strategy for operation, maintenance, and enhancements to improve reliability, minimize risk, and meet projected infrastructure demands.

The needs and priorities reflected in the Campaign Plan are a direct result of the recognition that in order to deliver reliable support to the Nation and Armed Forces, analytical and decision-making processes must explicitly address the risks and uncertainties associated with decisions and actions.

Risk and decision analysis can be implemented successfully through hard work.

Despite the desire to have it otherwise, there is no quick and easy path to risk-informed decision making. The approaches described within this report can be used to support decision-making activities and enhance the credibility and reliability of agency decisions. However, there are many challenges to applying these approaches, all of which can be overcome. A

few of the challenges that are likely to confront those who apply these methods include:

- *Decision framing*: Although probabilistic risk and decision analysis methods can usually be applied without necessarily altering the way that problems have traditionally been framed within the agency, application of rational decision theory may sometimes suggest deviations from the way that problems have been framed in the past. A transition to the use of probabilistic risk and decision analysis methods will be facilitated by opening one's mind to new ways of thinking about decision problems.
- *Ambiguity*: Decision theory and decision analysis assume a single decision maker who knows his objectives and preferences or values. Usually, however, the analyst is not the decision maker and there is ambiguity about objectives and preferences. For example, this may manifest itself in not knowing what objective function to use. This report has discussed sensitivity analysis as a way of overcoming this type of ambiguity. Sometimes there is more than one decision maker or there are multiple stakeholders. If so, the presence of different objectives or values may complicate an analysis. Normative economic and decision theory is fuzzy about how such groups should solve decision problems, or what choices are optimal in that situation. A practical decision analysis approach is to treat the group as a single decision maker and conduct sensitivity analysis on the parameters that describe decision maker's preferences parametrically.
- *Multi-objective problems*: Many agency problems involve multiple objectives. For example, agency personnel routinely address the need to make trade-offs among economic, human safety, and environmental outcomes. More often than not, these trade-offs are not considered explicitly. While not required for decision making under uncertainty, an openness to consider multiple objectives explicitly when framing decisions may help to resolve some of the ambiguities decision makers face.
- *Politics*: Policy is about how a pie should be distributed. Politics is about determining who gets the biggest piece. While policy can inform the decision process by clarifying decision objectives, politics will tend to contaminate the decision process and be an impediment to rational decision making if participants have individual agendas or priorities. Politics can seep into collaborative decision processes; for example, those that involve multiple levels of government (federal, state, and

local) or multiple agencies. The decision-making methods discussed in this report assume that analysts and decision makers are disinterested and objective participants in the process and there is a common objective. Decision analysis methods may be difficult to implement in a decision environment in which politics plays a significant role and participants are working to advance their own interests or the interests of the organizations that they represent.

- *Iterative vs. streamlined decision processes*: Streamlined decision-making processes that are implemented on tight schedules create the aura of efficiency. However, it is difficult to engage in an iterative decision process when working in such an environment because one lacks the freedom to explore the decision problem. This is not to say that probabilistic decision analysis cannot be applied in streamlined environments. Rather, if exploration of the decision problem is important - as would be the case in a transition toward the implementation of probabilistic decision methods in operations or planning - decision analysts may need freedom from constraints that might otherwise thwart an exploration of the decision problem.

Addressing these challenges is beyond the scope of this report. However, the authors propose that the concepts and approaches described offer the means for facilitating open and explicit exchange of ideas on these and other challenges involved in making risk-informed decisions.

Advancing analytical and decision-making practices

Advancing practices in USACE to bring analytical and decision-making practices in line with probabilistic risk and decision analysis principles will require commitment, at all levels, to the following principles:

- *Risk-informed decision making is based upon a comprehensive analysis of risks and uncertainties.* An analysis is comprehensive when it is responsive to the broad range of issues, concerns, and outcomes of interest to decision makers and stakeholders interacting through the deliberative process that informs decisions. Uncertainties that can affect the decision must be respected and, to an extent that it is consistent with the scope of decision making, their influence on outcomes must be quantified through rigorous analysis. Explicit consideration and analysis of uncertainty requires a commitment to transparency in the analytic-deliberative process as well as investments in time and other resources.

- *Deliberation is essential to decision making under uncertainty.* Effective communication about risks and uncertainties within USACE and with stakeholders and the public are keys to successful deliberation. While the Corps has made substantial progress in recent years in the areas of outreach and stakeholder engagement, implementing risk-informed decision making will require new commitments, standards, and approaches for the conduct of deliberation. It is arguably as important to give careful attention, through analysis, to the values, risk attitudes and perceptions germane to the decision problem under consideration, as it is to give attention to the science and engineering of the risk.
- *Advancing practice requires a commitment to change, experimentation, and learning.* Change is rarely easy. In fact, when it comes to changing long-held practices, organizational and individual resistance to change is to be expected. For these reasons, the commitment to advancing practices to bring them in line with risk-informed decision making must be a long-term commitment. USACE must be willing to experiment with new approaches, learn from those experiences, and carefully consider the lessons learned (both the failures and successes) as new standards of practice are established.

Risk-informed decision making offers the means for developing more resilient systems that provide long-term risk reduction benefits to the Nation. A commitment to developing and implementing such an approach will produce a stronger and more capable organization, better decisions, and superior service to the country.

References

Ascough, J.C., H.R. Maier, J.K. Ravalico, and M.W. Strudley. 2008. Future research challenges for incorporation of uncertainty in environmental and ecological decision-making, *Ecological Modelling* 219(2008): 383-399

Bedford, T., and R. Cooke. 2001. *Probabilistic risk analysis: Foundations and methods.* Cambridge, UK: Cambridge University Press.

Casman, E.A., M. G. Morgan, and H. Dowlatabadi. 1999. Mixed levels of uncertainty in complex policy models, *Risk Analysis* 19(1):33-42.

Clemens, R.T. 1996. *Making hard decisions.* Belmont, CA: Duxbury Press, 664.

De Finetti, B. 1974. *Theory of probability.* New York: Wiley.

Dempster, A.P. 1968. A generalization of Bayesian inference. *Journal of the Royal Statistical Society,Series B* 30: 205-247.

Fischer, K. 2004. *Value functions vs. utility functions.* Working Paper RP0416. Birmingham, UK: Aston Academy for Research in Management, Aston Business School.

Fischoff, B., P. Slovic, S. Lichtenstein, S. Read, and B. Combs. 1978. How safe is safe enough? A psychometric study of attitudes towards technological risks and benefits. *Policy Sciences* 8(1978): 127-158.

Garvey, P. 2008. Analytical methods for risk management: A systems engineering perspective. Boca Raton, FL: CRC Press, 264

Hobbs, B., P.T. Chao, and B.N. Venkatesh. 1997. Using decision analysis to include climate change in water resources decision making. *Climatic Change* 37(1997): 177-202.

Howard, R.A., and J.E. Matheson. 1984. *Readings in the principles and practice of decision analysis.* Menlo Park, CA: Strategic Decision Systems.

Howard, R.A. 1988. Decision analysis: Practice and promise, *Management Science* 34(6): 679-695.

Howard, R.A., J.E. Matheson, and D.W. North. 1972. The decision to seed hurricanes. *Science* 176(16 June): 1191-2002.

Howard, R.A. 2007. The foundations of decision analysis revisited, Chapter 2 in *Advances in decision analysis*, ed. W. Edwards, R.F. Miles, and D. von Winterfeldt, 621. Cambridge, UK: Cambridge University Press.

Kahneman, D., and A. Tversky. 1979. Prospect theory: An analysis of decision under risk, *Econometrica* 47: 263-291.

Kaplan, S., and B.J. Garrick. 1981. On the quantitative definition of risk, *Risk Analysis* 1(1): 11-27.

Keeney, R., and H. Raiffa. 1993. *Decisions with multiple objectives: Preferences and value tradeoffs.* Cambridge, UK: Cambridge University Press, 569.

Keeney, R., and D. von Winterfeldt. 2008. Practical value models, Chapter 13 in *Advances in decision analysis*, ed. W. Edwards, R.F. Miles, and D. von Winterfeldt, 621. Cambridge, UK: Cambridge University Press.

Lund, J.R. 2008. A risk analysis of risk analysis. *Journal of Contemporary Water Research and Education* 140(September):53-60.

Markowitz, H.M. 1952. The utility of wealth. *Journal of Political Economy* 60(1952):151-158.

Miles, R.F. 2007. The emergence of decision analysis, Chapter 1 in *Advances in decision analysis*, ed. W. Edwards, R.F. Miles, and D. von Winterfeldt, 621. Cambridge, UK: Cambridge University Press.

Morgan, M.G., and M. Henrion. 1990. Uncertainty: A guide to dealing with uncertainty in quantitative risk and policy analysis. Cambridge, UK: Cambridge University Press, 332.

Nau, R. 2007. Extensions of the subjective expected utility model, Chapter 14 in *Advances in decision analysis*, ed. W. Edwards, R.F. Miles, and D. von Winterfeldt, 621. Cambridge, UK: Cambridge University Press.

Pate-Cornell, M.E., and R.L. Dillon. 2006. The respective roles of risk and decision analyses in decision support. *Decision Analysis* 3(4): 220-232.

Prato, T. 2005. Bayesian adaptive management of ecosystems. *Ecological Modelling* 183(2005): 147-156.

Saaty, T.L. 1980. *The analytic hierarchy process.* New York, NY: McGraw Hill, p. 287.

Savage, L.J. 1954. *The foundations of statistics.* New York, NY: Wiley.

Shafer, G. 1976. *A mathematical theory of evidence.* Princeton, NJ: Princeton University Press

Slovic, P. 1987. Perception of risk. *Science* 236(17 April):280-285.

Starr, C. 1969. Social benefit versus technological risk. *Science* 165(19 September):1232-1238.

Tversky, A., and D. Kahneman. 1974. Judgment under uncertainty: Heuristics and biases. *Science* 185(27 September):1124-1131.

Tversky, A., and D. Kahneman. 1992. Advances in prospect theory: Cumulative representation and uncertainty. *Journal of Risk and Uncertainty* 5: 297-323.

von Neumann, J., and O. Morgenstern. 1947. *The theory of games and economic behavior.* Princeton, NJ: Princeton University Press.

von Winterfeldt, D., and W. Edwards. 2007. Defining a decision analytic structure. Chapter 6 in *Advances in decision analysis*, ed. W. Edwards, R.F. Miles, and D. von Winterfeldt, 621. Cambridge, UK: Cambridge University Press.

Yokota, F., and K.M. Thompson. 2004(a). Value of information literature analysis (VOILA): A review of applications in health risk management. *Medical Decision Making* 24(3):287-298.

Yokota, F., and K.M. Thompson. 2004(b). Value of information analysis in environmental health risk management decisions: Past, present, and future. *Risk Analysis* 24(3):635-650.

Zadeh, L.A. 1965. Fuzzy sets. *Information and Control* 8(3):338–353.

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14. ABSTRACT

This report introduces concepts, principles, and approaches for addressing uncertainty in decision making. The sources of uncertainty in decision making are discussed, emphasizing the distinction between uncertainty and risk, and the characterization of uncertainty and risk. The report provides a brief overview of decision theory and presents a practical method for modeling decisions under uncertainty and selecting decision alternatives that optimize the decision maker's objectives. The decision modeling methods introduced in this paper are suitable for both data rich and data poor decision environments. This report describes how to analyze the sensitivity of a decision model to improve understanding of the decision problem and build confidence in the conclusions of an analysis. Principles of adaptive management and adaptive engineering are discussed from a decision analysis perspective. Examples are provided to demonstrate how these methods could be applied within the U.S. Army Corps of Engineers.

15. SUBJECT TERMS

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